Not-So-Cleansing Recessions*

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Abstract

Recessions are periods in which the least productive firms in the economy exit, and as the economy recovers, they are replaced by new and more productive entrants. These *cleansing effects* improve the average firm productivity. At the same time, recessions induce a loss of varieties. We show that the long-run welfare effects trade off these two forces. This trade-off is governed by *love-of-variety* and the elasticity of substitution in aggregate production. If industry output is aggregated with the standard CES aggregator, recessions do not bring about any improvement in GDP and welfare. If the economy features more *love-of-variety* than CES, the social planner optimally subsidizes economic activity both in steady state and even more so in recessions to avoid firm exit. We use the model and quasi-exogenous variation in demand to estimate love-of-variety. We find it to be significantly higher than implied by CES aggregation, suggesting that even the long-run effects of recessions are negative. Finally, we quantitatively characterize the optimal policy response both along the transition and in the steady state.

Keywords: cleansing effects of recessions, business cycle, love-of-variety.
JEL Codes: E32, D31, L11

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This is really at the bottom of the recurrent troubles of capitalist society. They are but temporary. They are the means to reconstruct each time the economic system on a more efficient plan. But they inflict losses while they last, drive firms into the bankruptcy court, throw people out of employment, before the ground is clear and the way paved for new achievement of the kind which has created modern civilization and made the greatness of this country.

— Joseph A. Schumpeter *on the Great Depression, 1934*

1 Introduction

Recessions are often periods of increased reallocation of economic activity. The *liquidationist* view of business cycles has long posited that during downturns, unproductive economic units exit, and, as the economy recovers, they are replaced by new and more productive firms. Stiglitz (1993) refers to this effect as the *"silver lining of economic recessions"*. The intellectual roots of this argument can be traced back to Schumpeter (1934), who viewed recessions as a necessary short-term pain through which the economy improves its long-run outcomes.

Figure 1 shows the evolution of the number of active firms in the administrative data from Spain (SABI). Comparing the data with a linear trend estimated until the period before the 2007-08 recession, we see a large and persistent drop in the number of firms. Figure 2, based on the same data, shows that while in *normal times* the estimated TFP distribution of entering and exiting firms is very similar, in recession periods this does not hold. In particular, we find that entrants are substantially more productive than exiters during the recession. This represents prima facie evidence of a positive reallocation during recessions: exiting firms are replaced by, on average, better ones, consistent with the creative destruction in Schumpeterian theories. However, this reallocation may not necessarily induce output or welfare improvements even in the long run, when the recession has subsided.

In this paper, we revisit the role of recessions as moments of *cleansing* and creative destruction and, in particular, their long-run effects on GDP and welfare. We consider a parsimonious model of firm dynamics with heterogeneous firms à la Hopenhayn (1992)-Melitz (2003) and study the cleansing effect of business cycles driven by fluctuations in the fixed costs of production. In our model, temporary fluctuations can permanently change the firm size distribution, generating history dependence. In this context, we show that increases in fixed costs have cleansing effects through Schumpeterian forces: after a recession, fewer firms operate, and they are, on average, more productive than before the downturn. However, these productivity gains do not necessarily translate into higher long-run GDP and welfare.

When products are differentiated, the output and welfare effects of the number of varieties and average productivity gains are governed by different forces. Changes in the number of firms translates into output and welfare gains according to the *love-of-variety* (LoV) of the aggregator. Conversely, improvements in the average productivity of firms affects the economy through the elasticity of demand. We highlight this trade off in an economy with the original

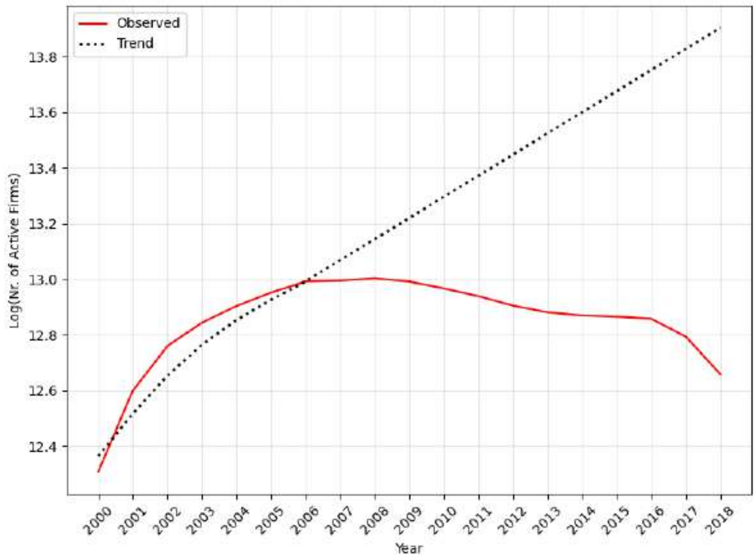

Figure 1: Number of Firms in Spain

Dixit and Stiglitz (1975) aggregation, where *love-of-variety* q and the elasticity of substitution σ are governed by different parameters.[1] We start by considering a partial equilibrium version, in which we show that the long-run welfare effect of recession is positive if and only if $q < 1/(\sigma - 1)$ while it is negative if $q > 1/(\sigma - 1)$. The classical Dixit and Stiglitz (1977) CES aggregation represents a special case of our setting, where $q = 1/(\sigma - 1)$, in which the long-run effect of recessions is exactly zero. In this knife-edge case, the effect of the variety loss and the average productivity gains exactly offset each other as they are governed by the same elasticity $1/(\sigma - 1)$. We conclude that, in this setting, *cleansing effects* are neither necessary nor sufficient to determine the long-run welfare effects of business cycles.

Next, we show that this intuition extends to general equilibrium with a minor caveat: in GE, the cleansing effect frees up labor resources. After a recession, the economy features fewer, more productive firms. As a consequence, less of the finite labor endowment is used to pay the fixed costs of production. This implies that fixed cost cycles induce higher long-run welfare even in CES economies. Nonetheless, we show that for any fixed cost cycle there exists a unique level of LoV $q^\star$, such that long-run GDP and welfare are identical to their pre-cycle levels.

We conclude our theoretical analysis with two normative results. We show that the market allocation is constrained efficient if and only if the economy features CES aggregation. This extends results by Dixit and Stiglitz (1977) and Dhingra and Morrow (2019) to economies that also consider the presence of incumbent firms. We then show that when the economy values varieties more or less than implied by CES, the market equilibrium features either too few or too many firms, and characterize the optimal policy intervention. The planner can correct this steady-state inefficiency with a single instrument that changes the relative entry and fixed costs.

Importantly, we show that in economies in which output is aggregated with stronger love-for-

[1] See Brakman and Heijdra (2001) for a reprint of Dixit and Stiglitz (1975). See also Ethier (1982) and Benassy (1996) for similar aggregation.

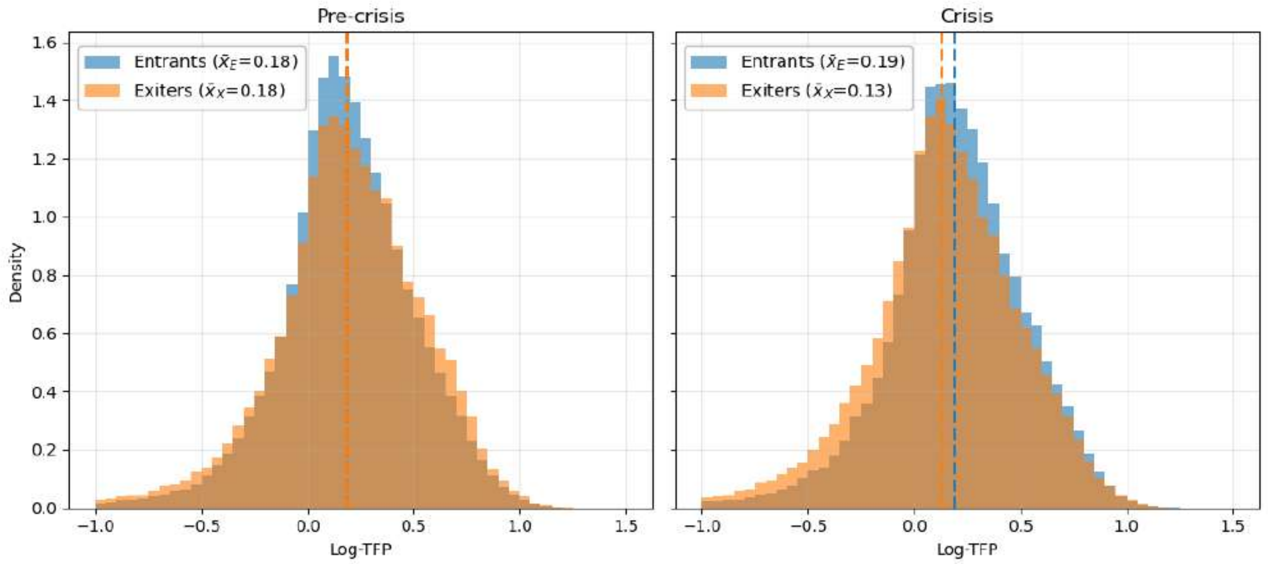

Figure 2: Distribution of TFP - Entrants vs Exiters. We test $H_0 : \bar{x}_E - \bar{x}_X = 0$ at the 5% significance level. In the period preceding the Global Financial Crisis, we fail to reject H_0, while during the crisis, we reject H_0.

variety than in the CES benchmark, under mild assumptions on the productivity distribution, the social planner finds it optimal to increase fixed costs subsidies during recessions. This policy is motivated by inducing a smaller amount of exit during bad times than implied by the market equilibrium. This result speaks to frequent efforts by governments to reduce churn during bad times, as seen during the Covid-19 recession.[2]

Both our positive and normative analyses highlight the importance of the parameters governing love of variety q and the elasticity of demand σ. To inform our quantitative evaluation, in Section 5, we set out to estimate these key parameters. We do so using two equilibrium relations in our model. First, in equilibrium σ uniquely determines the profit rate of the economy. We use the WIOD data (Timmer et al., 2015) to obtain the distribution of profit rates for the 56 industries in the sample across 43 countries.[3] We find a median elasticity of substitution of 5.8, associated with a profit rate of 17.2%. To estimate q, we make use of another structural relation in our model: love-of-variety governs the extent of external returns in our economy. We show that, in our setting, the elasticity of output to an increase in downstream expenditure is equal to $1 + q$, where the "1" obtains from our constant technological returns to scale assumption. We use the quasi-exogenous shifts in foreign income from the instrumental variable developed by Ferrari (2024). This instrument combines shifts in destination markets for WIOD industries via their network exposure shares. We obtain an estimate of love-for-variety q between .51 and .63, depending on the specification. Comparing these estimates with the love-of-variety in CES implied by our distribution of σ, we conclude that all but 2 out of the 56 industries in our sample have a stronger love-of-variety than implied by CES. The direct consequence of our

[2]See Kozeniauskas et al. (2022) for an empirical evaluation of such programs.

[3]WIOD reports capital expenses together with economic profits, we, therefore, make the conservative assumption that all capital expenses are, in fact, profits, leading to an upward bias of the profit rate and a downward bias in σ.

empirical finding is that, for most of these industries, recessions are costly even in the long run, despite their cleansing effects on the productivity distribution.

We conclude by studying a quantitative version of our model. This allows us to evaluate the overall effect of recessions, accounting for transitional dynamics. We use our estimated parameters for love-of-variety and the elasticity of substitution, and calibrate the rest of the model to moments of the Spanish firm-level administrative data. First, we show that, given the estimated love-of-variety and elasticity of substitution, the planner would like to introduce a steady-state subsidy to the fixed cost. As highlighted by our normative results, the economy features too few firms, and the planner wants to induce a higher number of varieties. Quantitatively, we find that this subsidy can increase welfare by 55.30% in consumption equivalent variation (CEV). Next, we study business cycles in our quantitative model. We engineer a temporary increase in the fixed cost so that in the trough, 20.44% of firms exit, as in Spain in 2007-09. Absent any policy intervention, this recession induces a welfare loss of -5.28% CEV. This loss is largely driven by a very persistent loss of varieties (-6.5% after 25 years), consistent with the deviation from trend observed in the data. When we allow the social planner to intervene, it does so by subsidizing firms during the recession to minimize the exit of varieties. The planner fully shifts the absorption mechanism of the recession: it induces firms not to exit and reallocates labor from production to fixed costs payments. This policy makes the effect of the recession large but very short-lived and eliminates any long-run effect. This active budget-neutral policy reduces the welfare cost of the recessions by 34%, to -3.48% CEV. This result underscores the importance of the intervention in reducing the cost of business cycles.

Related Literature This paper draws insights from the literature on the optimal number of varieties (Dixit and Stiglitz, 1977; Dhingra and Morrow, 2019) to discuss the cleansing effects of recessions.[4] We build on Jovanovic (1982), Hopenhayn (1992), and Melitz (2003) to study the effect of recessions in economies with love-of-variety and firm dynamics. The notion that recessions can generate long-run benefits thanks to an acceleration of creative destruction dates back to Schumpeter (1934, 1939) and has been formalized in Caballero and Hammour (1994). They study an embedded capital model in which recessions induce firm exit, thereby accelerating the speed of modernization of installed capital. We study an economy in which there is no capital, but producers are heterogeneous. During recessions, the least productive firms exit and are subsequently replaced by entrants with higher average productivity. Importantly, we show that this is not enough to infer the behavior of output or welfare, as the equilibrium number of firms drops after a recession. The patterns predicted by our model are consistent with the large empirical evidence on the procyclical properties of entry and exit, both at the firm and product level (Dunne et al., 1989; Davis and Haltiwanger, 1992; Broda and Weinstein, 2010; Kehrig, 2015; Lee and Mukoyama, 2015; Argente et al., 2018; Tian, 2018; Argente et al., 2024).

Theoretically, our work is related to the literature on the aggregate consequences of en-

[4] This problem has been the subject of a large literature which includes Spence (1976); Dixit and Stiglitz (1977); Mankiw and Whinston (1986); Zhelobodko et al. (2012); Dhingra and Morrow (2019); Matsuyama and Ushchev (2020). See Brakman and Heijdra (2001) for an early review.

try, exit, and firm heterogeneity, such as Chatterjee and Cooper (1993); Bilbiie et al. (2012); Clementi and Palazzo (2016); Bilbiie et al. (2019); Carvalho and Grassi (2019); Bilbiie and Melitz (2020); Gouin-Bonenfant (2022); Ferrari and Queirós (2024); Bendetti-Fasil et al. (2024); Collard and Licandro (2025), and the literature on external effects in macroeconomics following Baxter and King (1991). Related to our conclusion, Hamano and Zanetti (2017, 2022) highlight the existence, in a CES economy, of the welfare tradeoff between higher average productivity and fewer available varieties. We show that for a CES economy, the two effects perfectly offset each other in PE. In GE, the only welfare effect is the labor savings and the income windfall that this generates. The tradeoff between better firm selection and loss of variety becomes welfare-relevant only away from CES. In particular, when love-of-variety is larger than implied by the CES benchmark. Barlevy (2002), Ouyang (2009), Moreira (2016), and Acabbi et al. (2022) suggest that the cleansing effects may be lower than expected. Barlevy (2002) and Acabbi et al. (2022) argue that procyclical job match quality dominates the cleansing role of recessions. Ouyang (2009) considers a setting in which recessions may halt the entry of very productive firms and, thereby, reducing long-run growth. Moreira (2016) shows that firms born in recessions remain smaller than their counterparts born in good times. Our main result is related but distinct: even when measured productivity improves during recessions, this may induce a loss of varieties that manifests in lower GDP and welfare.

Finally, our paper is closely related to Ardelean (2006), Baqaee et al. (2023), and Gaudio and Poilly (2025), who provide estimates for love-of-variety. Using firm-to-firm transactions, Baqaee et al. (2023) estimate the elasticity to new suppliers to be .3. Using establishment entry, Gaudio and Poilly (2025) estimate an elasticity of .49. Our estimates based on foreign demand shocks suggest that the aggregate love-of-variety effect is approximately .5-.6, broadly consistent with the range implied by Baqaee et al. (2023) and Gaudio and Poilly (2025), and significantly larger than implied by a CES aggregator.

2 An Industry Model of Entry and Exit

We study a standard model of firm dynamics (Hopenhayn, 1992; Melitz, 2003). The key difference is that we allow the elasticity of demand for differentiated varieties to differ from the love-of-variety effect. We show that this carries important implications for the cleansing effects of recessions. We begin by describing the economic environment, specifying the structure of production and consumption in the economy. To isolate the key economic forces, we start by analyzing a partial equilibrium setting and the move to general equilibrium.

2.1 Setup

Production We consider an industry in monopolistic competition, in which firms hire labor to produce their variety with constant returns to scale in labor l and heterogeneous productivity z: $y(z) = zl$. Productivity is distributed according to some density function m. To produce, firms must pay a fixed cost of f^c labor units. In this partial equilibrium context, we consider a small industry in isolation, so that labor can be hired at a wage w, which is not affected by the industry. We set labor as the numeraire good: $w = 1$. Firms maximize their profits $\pi(z) = y(z)(p(y) - 1/z) - f^c$ by choosing their price, subject to their demand constraint.

Industry Output A perfectly competitive intermediary combines varieties and sells the final composite good to households. The intermediary aggregates with a generalized CES production function à la Dixit and Stiglitz (1975), Ethier (1982), Benassy (1996)

$$Y = M^{q-\frac{1}{\sigma-1}} \left[\int y(z)^{\frac{\sigma-1}{\sigma}} m(z) dz \right]^{\frac{\sigma}{\sigma-1}}. \tag{1}$$

We follow Hopenhayn (1992) and define $m(z)$ as a density function. The total number of varieties M is given by $\int m(z)\,\mathrm{d}z$. As a notational convention, we use $m(z)$ when referring to density functions, and $\mu(z)$ when referring to probability density functions that integrate to 1, i.e. $\mu(z) := m(z)/M$. Note, further, that $m(z)$ is the distribution of productivity of active firms, which is determined in equilibrium.

Two key parameters govern the aggregator: the elasticity of substitution, σ, which measures product similarity, and love-of-variety (LoV). Following Benassy (1996), we define LoV as the "gain derived from spreading a certain amount of production between M differentiated products instead of concentrating it on a single variety". We highlight in Remark 1 that in our aggregator, LoV is exactly equal to q. LoV measures the additional productivity the intermediary derives from having a larger number of available varieties. This effect constitutes an aggregate production externality.[5] The classic Dixit and Stiglitz (1977) CES-aggregator is a special case where $q = q^{CES} := 1/(\sigma - 1)$. Hence, $q > 1/(\sigma - 1)$ implies stronger LoV than in the CES case and vice versa.[6]

Remark 1. *The aggregator in eq.* (1) *exhibits love-of-variety equal to q.*

All proofs are relegated to Appendix B.

The aggregator in (1) satisfies a number of desirable properties, striking a balance between the tractability of CES aggregation and the ability to govern separately *love-of-variety* and the price elasticity of demand. We come back to the importance of the functional form assumption for our results at the end of this section. In Appendix A.1 we prove that in the broader class of homothetic preferences described in Matsuyama (2023), the aggregator in eq. (1) is the only one that jointly satisfies separability into an iso-elastic variety externality and an aggregator with zero LoV. In this sense, eq. (1) is the only aggregator in which the variety externality can be cleanly separated from demand aggregation.

[5]Our setting can be interpreted as having an intermediary downstream firm, with production function Y, purchasing its inputs from the upstream variety producers. Therefore, LoV in our model closely aligns with the LoV estimated in Baqaee et al. (2023). In an equivalent reading without intermediaries, the aggregator in (1) is the household utility function. The extra utility from having a larger number of available varieties is then an aggregate demand externality as in Blanchard and Kiyotaki (1987).

[6]Readers more familiar with the Melitz (2003) model aggregation can think of our aggregator as being $Y = M^{q+1} \left[\int y(z)^{\frac{\sigma-1}{\sigma}} \mu(z) dz \right]^{\frac{\sigma}{\sigma-1}}$, with $\mu(z) = m(z)/M$ being a probability density function.

Demand There is a representative household with utility $\mathcal{U}(Y)$, $\mathcal{U}' > 0$, and exogenous total income $\mathcal{I}$. We relax exogeneity when we consider a general equilibrium economy in Section 3. Households spend all their income on the composite good.

Entry and Exit Entry follows closes Hopenhayn (1992) and Melitz (2003): ex-ante identical entrepreneurs pay a fixed cost of entry f^e in terms of labor to enter and draw a productivity level z from a baseline probability distribution μ^E. Potential entrants form expectations on their post-entry profits when making entry decisions. We assume that firms cannot borrow against their future profits, which implies that firms with negative current profits exit the economy, even if the present discounted value of the firm is positive. We return to this assumption at the end of the section.

2.2 Equilibrium

The intermediary's problem implies a demand schedule for each variety, which is taken into account by the corresponding monopolist. The monopolistic competition solution is independent of the aggregate production externality and implies the usual constant-markup pricing $p(z) = \frac{\sigma}{\sigma-1}\frac{1}{z}$. At the optimum, firm profits, π, depend on the firm's productivity z and on competition, m. Notably, m only matters through an aggregate statistic: the index $\int z^{\sigma-1}m(z)\,\mathrm{d}z$, which we call *market intensity*. Profits are given by

$$\pi(z,m) = \frac{\mathcal{I}}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}m(z)\,\mathrm{d}z} - f^c. \tag{2}$$

The problem of potential entrants is to form expectations on the profits they would obtain upon entry. These profits depend on the makeup of firms already operating in the economy. We refer to these firms as *incumbents* and denote the distribution of their productivity with m_{t-1}. Potential entrants, knowing m_{t-1}, form expectations on profits taken over the probability distribution of productivity μ^E. Importantly, firms understand that their entry decision is simultaneous to that of other potential entrants. As a consequence, they do not just consider the competition given by current incumbents but rather what the market will look like once all potential entrants have entered.[7] We call this ex-post distribution m_t. Entrepreneurs choose to enter if $\mathbb{E}_{\mu^E}[\max\{\pi(z_t,m_t),0\}] - f^e > 0$. The presence of a fixed cost implies that some firms, after their productivity draw, may find it unprofitable to produce. This fixed cost implies the existence of a threshold productivity $\underline{z}_t$ such that $\pi(\underline{z}_t,m_t) = 0$. Entrants and incumbents alike produce if their productivity z lies above $\underline{z}$ and leave otherwise.

Denote the mass of entrants drawing by $E_t \geq 0$ so that the distribution of successful entrants is given by $m_t^E(z) = E_t\mu^E(z)\mathbb{I}(z \geq \underline{z})$. After entry, the distribution of firms, m, is given by the sum of surviving entrants and incumbents in eq. (3). Equilibrium is reached if no additional entrepreneurs want to enter (eq. 4) and no additional incumbents or recent entrants want to quit (eq. 5); the two equilibrium objects – cutoff, $\underline{z}_t$ and mass of entrants, E – adjust to jointly

[7] We prove in Appendix A.2 that this is equivalent to the limit of an iterative entry game.

satisfy eqs. (4) and (5):

$$m_t(z) = m_{t-1}(z)\mathbb{I}_{\{z\geq \underline{z}_t\}} + \overbrace{E_t\mu^E(z)\mathbb{I}_{\{z\geq \underline{z}_t\}}}^{m_t^E(z)}, \quad (\forall z \geq 0), \tag{3}$$

$$\mathbb{E}_{\mu^E}[\max\{\pi(z, m_t), 0\}] \leq f^e, \tag{4}$$

$$\pi(\underline{z}_t, m_t) = 0. \tag{5}$$

Equations (3-5) characterize an equilibrium. We define a steady-state equilibrium as an equilibrium where eq. (3) has a fixed point.

Definition 1 (Steady-State Equilibrium). *A steady-state equilibrium in this economy is an allocation satisfying eqs. (4) and (5) and where eq. (3) is such that* $m_t(z) = m_{t-1}(z)$, $\forall z \geq 0$.

Since there is no exogenous exit in the economy, by Definition 1, in steady state, there is no entry: $E = 0$. Outside of steady state, if there is scope for entry, $E > 0$ and eq. (4) holds with equality. Combining (2), (4), and (5), we obtain the following equilibrium conditions:

$$E = \frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}_t^{\sigma-1}}{\int_{\underline{z}_t} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z} - \frac{\int_{\underline{z}_t} z^{\sigma-1}m_{t-1}(z)\ \mathrm{d}z}{\int_{\underline{z}_t} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z}, \tag{6}$$

$$\frac{f^e}{f^c} = \int_{\underline{z}_t}^{\infty}\left[\left(\frac{z}{\underline{z}_t}\right)^{\sigma-1} - 1\right]\mu^E(z)dz. \tag{7}$$

Notably, the equilibrium conditions do not depend on q. LoV only affects the production of the downstream intermediary firm. This change in production does not translate into higher demand for the upstream firms, because total expenditure on the composite good is fixed to $\mathcal{I}$. Economies with higher q have larger final good output and lower price such that the revenue of the intermediary remains fixed at $\mathcal{I}$. Examining eq. (7), we establish the following result.

Lemma 1 (Cutoff determination). *Whenever there is entry into the economy* ($E > 0$)*, the productivity cutoff* $\underline{z}_t$ *is independent of the incumbent distribution* m_{t-1}.

To build intuition for Lemma 1, it is useful to note that the profits of a firm with productivity z in eq. (2) only depend on the market size $\mathcal{I}$, the productivity of the firm z and a single aggregate statistic: market intensity $\int z^{\sigma-1}m_t(z)$. This specific result is a consequence of important properties of CES and monopolistic competition. In general, the identity of the marginal firm depends on the state of competition in the economy. CES and monopolistic competition imply that competition is fully summarized by the market intensity or, equivalently, by the price index (see Matsuyama and Ushchev, 2017). Furthermore, entrants and incumbents enter symmetrically in the market intensity. This is because incumbents have no competitive advantage over new entrepreneurs. As a consequence, the exact composition of m is irrelevant conditional on a market intensity. Whenever there is a positive flow of new firms in the economy, the identity of the marginal firm is independent of the incumbents' distribution.[8]

[8] We refer to the discussion in the proof of Lemma 1 for a more in-depth description of how the CES-monopolistic competition framework generates this result.

This result underpins the behavior of the economy during business cycles. In this economy, the only dynamic variable linking periods is the incumbents' productivity distribution. Since the identity of the marginal firm is independent of it during economic expansions, the economy is *memoryless* during booms.

In contrast, if $E = 0$, the cutoff depends on the distribution of incumbents because if there is exit, then the marginal exiting firm is an incumbent. Equation (4) is slack and eq. (5) rearranges to $\pi(\underline{z}_t, m_{t-1}(z)\mathbb{I}_{\{z \geq \underline{z}_t\}}) = 0$. Therefore, the incumbents' distribution determines the cutoff during recessions, which implies that the economy features *history-dependence* in busts.

This completes the characterization of the partial equilibrium of this economy. Next, we study the effect of business cycles.

2.3 Business Cycles

Since we are interested in isolating the long-run consequences of temporary fluctuations, we focus on steady states. We return to the full dynamics at the end of the section.

We define business cycles as one-time, unexpected shocks to the fixed costs of production, f^c. Such cycles could, for example, be driven by varying financing conditions in working capital constraints.[9] We focus on this kind of recession as they have the highest potential to generate *cleansing effects* since fixed costs directly govern selection. We consider alternative sources of business cycle fluctuations in Section 2.4.

Formally, consider an economy running through three phases, capturing the dynamics of crises. In phase $\tau = 1$, fixed costs equal f_l^c, then unexpectedly increase to $f_h^c > f_l^c$ in phase $\tau = 2$, and subsequently revert to f_l^c in phase $\tau = 3$; subscripts l and h refer to low and high.[10]

In phase 1, firms enter until the industry reaches the equilibrium determined by eqs. (6) and (7). In phase 2, the rise of fixed costs forces some of the incumbents to exit. Since all incumbents entered in phase one, the distribution at the beginning of phase 2 is given by $m_1 = E_1\mu^E(z)\mathbb{I}_{\{z \geq \underline{z}_1\}}$. The rise of the fixed cost implies that the productivity cutoff, determined by the zero-profit condition alone (eq. 5), increases to $\underline{z}_2 > \underline{z}_1$, generating the new distribution $m_2 = m_1(z)\mathbb{I}_{\{z \geq \underline{z}_2\}}$. We show this process in panels A and B of Figure 3. In phase 3, the fixed costs revert to f_l^c. Survivors of the crisis are now incumbents. Additionally, new firms enter. Equations (6) and (7) hold, and $E_3 > 0$. Panel C of Figure 3 shows the post-recession distribution m_3. Note that the productivity of the least productive active firm is the same before and after the cycle, since by Lemma 1, the cutoff $\underline{z}_3$ equals the pre-crisis cutoff $\underline{z}_1$. Yet, the distribution of firms differs and $m_1 \neq m_3$ due to the presence of incumbents.

This surprising result can be understood by noting that in this model, there is an important asymmetry between entry and exit. Entry occurs under the veil of ignorance: firms do not know their productivity when they choose to enter. On the other hand, exit occurs when firms already

[9] An alternative interpretation of these shocks is that the fixed costs are produced in a different sector c from labour and that downturns are triggered by reductions in the TFP of sector c.

[10] Note that considering the phase 3 reversion to f_l^c is equivalent to studying the limit of a slow-moving process of mean-reversion of the fixed cost to the long-run mean of f_l^c. We characterize this problem in Appendix A.6.

know their type z, which implies that exit is *selected*. In other words, entry occurs along the entire support above the cutoff $\underline{z}$, while exit always happens at the bottom of the distribution.

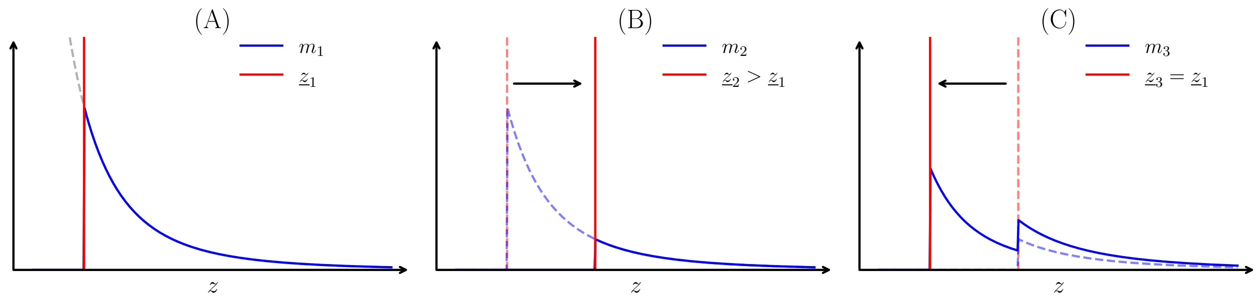

Figure 3: The figure shows the entry and exit dynamics over the business cycle. Panel (A) shows the distribution m_1 before the shock hits. Upon impact, the left tail of firms with productivity less than $\underline{z}_2$ leave, creating distribution m_2 (B). Finally, after fixed costs return to pre-shock levels and new firms drawn from the baseline distribution (μ^E) enter, m_3 becomes the distribution of productivities in the market (C). The dashed light-blue line refers to $m_{\tau-1}$.

To characterize the behavior of output and welfare, we can substitute the equilibrium production choice of each firm into the aggregator in eq. (1), to obtain aggregate output (GDP)

$$Y_\tau = M_\tau^{q-\frac{1}{\sigma-1}} L_\tau^p \left(\int z^{\sigma-1} m_\tau(z) \, \mathrm{d}z \right)^{\frac{1}{\sigma-1}}. \tag{8}$$

Where, L_τ^p is the amount of labor used in production, and $\left(\int z^{\sigma-1} m_\tau(z) \, \mathrm{d}z \right)^{\frac{1}{\sigma-1}}$ is aggregate technical efficiency, which we refer to as TFP. Proposition 1 characterizes how business cycles affect aggregate output relative to pre-cycle values. Define $\Delta \log X \equiv \log X_3 - \log X_1$.

Proposition 1 (Cleansing Effects of Cycles)**.** *The change in output after the crisis is given by*

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1} \right) \Delta \log M + \frac{1}{\sigma - 1} \Delta \log \left(\int z^{\sigma-1} m(z) \, \mathrm{d}z \right). \tag{9}$$

The recession reduces the number of firms but leaves aggregate productivity unchanged:

$$\Delta \log M < 0 \text{ and } \Delta \log \left(\int z^{\sigma-1} m(z) \, \mathrm{d}z \right) = 0.$$

As a consequence, GDP and welfare increase if and only if the economy values varieties less than in CES aggregation:

$$\Delta \log Y \gtreqless 0 \Leftrightarrow q \lesseqgtr q^{CES}. \tag{10}$$

Proposition 1 is our first main result: whether long-run GDP and welfare are larger than before the recession depends uniquely on the LoV parameter q. In the long run, an economy that undergoes a temporary increase in the fixed cost of production has fewer but, on average, more productive firms. This result comes with several additional important observations. First, our result refers exclusively to long-run effects: any potential welfare loss is not driven by the transition in phase 2. When $q < q^{CES}$, the economy might still be worse off once transition costs are accounted for. What we show is that if $q > q^{CES}$, long-run welfare is lower even without the transition costs. Second, this negative long-run effect arises even if all the classic Schumpeterian forces of recessions are present: in our model, new entrants are, on average, more productive than exiters during the cycle. Recessions induce a *selection effect*. Yet, they may still result in long-run welfare losses. The aggregate consequences of recessions trade off fewer firms, whose effect is governed by LoV q, with higher average productivity, whose effect is governed by the elasticity of demand and specifically $q^{CES} = \frac{1}{\sigma-1}$. Third, we remark that the CES case is a knife-edge parametric restriction where these two coincide:

Remark 2 (CES Aggregation)**.** *In the special case of CES aggregation* $\left(q = q^{CES}\right)$*, long-run output and welfare are unchanged pre- and post-recession:* $\Delta \log Y = 0$.

To build intuition for the result, it is useful to restate the production choice of an individual firm with productivity z, and the associated profits

$$y(z) = \mathcal{I}\frac{\sigma-1}{\sigma}\frac{z^{\sigma}}{\int z^{\sigma-1}m(z)\,\mathrm{d}z} \qquad\qquad \pi(z) = \frac{\mathcal{I}}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}m(z)\,\mathrm{d}z} \tag{11}$$

Given this policy function and payoff, it is clear that the sufficient aggregate statistic is *market intensity* $\mathcal{Z} = \int z^{\sigma-1}m(z)\,\mathrm{d}z$. Importantly, the composition of this statistic does not matter: a firm with productivity z would make the same output choice and obtain the same profits in two economies whose $\mathcal{Z}$ is the same, but one has many unproductive firms while the other has few very productive firms. Second, potential entrepreneurs make entry decisions equating expected profits to the entry cost f^e. During the recession, the market intensity drops as firms leave the economy. During the transition back to the steady state, firms enter until there are positive expected profits and drive market intensity up. Since the size of the industry $\mathcal{I}$ is constant and there is no strategic advantage of incumbents on entrants, the economy converges back to the original market intensity. However, the recession permanently changes the firms' distribution. Firms entering during the recovery are, on average, more productive than exiting firms. As a consequence, exiters are replaced less than 1-for-1, and the number of firms M drops.

In a CES economy, output is given by $Y = L^p \left(\int z^{\sigma-1}m(z)\,\mathrm{d}z\right)^{\frac{1}{\sigma-1}}$ and therefore depends uniquely on aggregate TFP and aggregate production labor, L^p. The latter, however, does not depend on the fixed cost cycle but exclusively on market size $\mathcal{I}$ and σ, as already argued in Section 2.2. Since aggregate TFP and aggregate labor used in production are unchanged, so is output and, therefore, welfare.

This effect is best understood considering eq. (8) and rewriting it in terms of the total number of firms and average productivity. Taking logs and differencing pre- and post-crisis values yields

$$\Delta \log Y = q\Delta \log M + \Delta \log L^p + \Delta \log \bar{z}, \tag{12}$$

where we call $\bar{z}_\tau := \left(\int z^{\sigma-1}\mu_\tau(z)\, \mathrm{d}z\right)^{\frac{1}{\sigma-1}}$ *average productivity*. The first term, $q\Delta \log M$, is the *variety effect*, the second term, represents changes in labor used for production, and the third term the *selection effect* of the crisis. The variety effect is driven by changes in the number of firms M and its effect on output is governed by LoV, q. The selection effect is channeled through changes in average productivity as exiting firms, on average, are less productive than entrants. This effect is governed by $\frac{1}{\sigma-1}$ since, noting that aggregate technical efficiency is unchanged immediately implies $\Delta \log \bar{z} = \frac{1}{\sigma-1}\Delta \log M$. Therefore, in PE, the variety effect dominates the selection effect for $q > q^{CES}$, while for CES economies, the two cancel out exactly.

Proposition 1 highlights another important aspect of our model, formalized in Remark 3.

Remark 3 (Path Dependence). *The stationary steady-state equilibrium is path-dependent.*

We considered economies that, before and after the recession, feature identical parameters. Nonetheless, they are characterized by different equilibrium allocations. This property is fully driven by the presence of incumbents. Our business cycles involve changes in the fixed costs of production, and the productivity cutoff is a direct function of these costs. Starting from a steady state with fixed cost, f^c, and distribution of active firms, m, a temporary increase in fixed costs truncates the entire distribution m at a higher cutoff during the transition. When fixed costs subsequently revert to f^c, and entry resumes at the original cutoff, the new entrant cohort is selected at the lower cutoff while the surviving incumbents come from the truncated m. Consequently, the composition in the new steady state differs, and a temporary recession permanently reshapes the firm distribution and, in turn, the steady-state allocation.

2.4 Alternative Sources of Business Cycle Fluctuations

In our analysis so far, we have considered a specific source of business cycle fluctuations: movements in the fixed operating cost. We interpret these costs broadly as stand-ins for operating capital constraints or quasi-fixed factors. Notably, these recessions have the highest chance of generating cleansing effects since they directly affect the selection margin. Here, we consider two alternative sources of fluctuations: aggregate TFP shocks and movements in the entry cost.

Aggregate TFP Cycles Consider an unexpected temporary decrease in aggregate productivity A, such that the productivity of all firms goes from z to $Az < z$. We characterize the behavior of our economy in Proposition 2.

Proposition 2 (Aggregate TFP Cycles). *Recessions driven by temporary decreases in aggregate TFP have no long-run effects.*

To understand this surprising result, note that, in our economy, temporary shocks have long-run effects only if they affect entry and exit decisions. An aggregate TFP shock moves the effective productivity of all firms equally. As a consequence, the relative productivity is unchanged. Entry/exit decisions might still be affected if the size of the industry changes. However, in this partial equilibrium setting the industry size is fixed at $\mathcal{I}$. Therefore, an aggregate TFP shock does not induce any cleansing effect in this economy. It does however affect output, which remains lower than its pre-crisis value as long as A remains lower than 1. Yet, as entry and exit are not affected, if A returns to its initial value, the economy returns to its original steady state.[11]

Entry Cost Cycles We study the effect of changes in the entry cost. To this end, we have to change our model to have a positive entry flow in steady state. Consider a version of our economy where firms exogenously exit at rate $\delta > 0$ as in Melitz (2003). Then the law of motion of the productivity distribution is given by $m_t(z) = [(1-\delta)m_{t-1}(z) + E_t\mu^E(z)]\mathbb{1}_{\{z \geq \underline{z}_t\}}$. This standard formulation implies that in the steady state, there is positive entry to exactly offset the churn due to the exit rate δ. Importantly, it also implies that the steady state is unique and history-independent. Denote $\underline{z}^{\text{ss}}$ and M^{ss} the steady-state cutoff and number of firms, respectively.

Suppose that the economy, starting from a steady state, experiences an unexpected increase in the entry cost f^E. We characterize the behavior of this economy in Proposition 3.

Proposition 3 (Entry Cost Cycles)**.** *Consider a temporary increase in the entry cost. Along this transition, the following holds*

a) The number of varieties converges to M^{ss} from below.

b) The average productivity of active firms converges to $\bar{z}^{\text{ss}}$ from below.

c) The temporary increase in the entry cost always generates welfare losses, independently of love-of-variety.

Proposition 3 characterizes the transitional behavior of the economy after a temporary increase in the fixed cost of entry f^e. First, when the entry cost is higher, the business dynamism of the economy is hampered. Throughout the transition, the number of firms is below its steady-state value, inducing variety losses. Unlike a fixed-cost recession, an increase in the entry cost is not *cleansing*. Along the transition, selection forces are weakened as potential entrants exert a smaller competitive pressure on unproductive incumbents. As a consequence, the average productivity of firms declines. Interestingly, output does not fall substantially on impact since the output losses are only driven by the steady erosion of the number of firms and the slow decline in average productivity. As the entry cost converges back to its original level, the economy returns to its steady state with a slow increase in the number of firms and average productivity. We conclude that a temporary increase in f^e induces aggregate behavior that is more reminiscent of *stagnation* episodes than of recessions.

Since both the number of firms and average productivity worsen along the transition, the welfare effects are unambiguously negative, independently of how the economy values varieties.

[11]This result extends to the general equilibrium model we consider in the next section under perfectly inelastic labor supply.

3 General Equilibrium

In this section, we extend our result to a general equilibrium setting. Relative to the model in Section 2, there are two changes. First, production labor demanded by firms now affects the market-clearing wage. Labor market clearing dictates:

$$L^p + Mf^c + Ef^e = \bar{L}. \tag{13}$$

Second, expenditure on the consumption good PY is equal to the revenues of the firms R, so that the budget constraint is $R = \bar{L} + \Pi$.[12] Then, setting labor as the numeraire ($w = 1$), firm profits become:

$$\pi(z, m) = \frac{R}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} m(z)\, \mathrm{d}z} - f^c. \tag{14}$$

Entry and exit are still pinned down by the *free-entry condition* and *zero-profit condition* (eqs. 4 and 5). When entry is strictly positive, eqs. (4) and (5) can be rearranged into a GE analog of eq. (6)

$$E = \frac{R}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E(z)\, \mathrm{d}z} - \frac{\int_{\underline{z}}^{\infty} z^{\sigma-1} m^I(z)\, \mathrm{d}z}{\int_{\underline{z}}^{\infty} z^{\sigma-1} \mu^E(z)\, \mathrm{d}z}, \tag{15}$$

and eq. (7), which is unchanged from PE. The equilibrium is fully characterized by eqs. (7), (13), and (15). When there is no scope for entry, then $E = 0$, the *free-entry condition* is slack, and the *zero-profit condition*

$$\frac{R}{\sigma} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1} m^I(z)\, \mathrm{d}z} - f^c = 0 \tag{16}$$

pins down the mass of active firms in the economy. The equilibrium is fully characterized by eqs. (13) and (16), and $E = 0$.

3.1 Business Cycles

We consider the same source of fixed cost variation and establish the effect on long-run output. The following result extends Proposition 1 to general equilibrium:

Proposition 4 (Cleansing Effects of Cycles)**.** *The change in output after the crisis is given by*

$$\Delta \log Y = \left(q - \frac{1}{\sigma - 1}\right) \Delta \log M + \Delta \log L^p + \frac{1}{\sigma - 1} \Delta \log \int z^{\sigma-1} m(z)\, \mathrm{d}z, \tag{17}$$

[12] We maintain that labor is inelastically supplied, an assumption we relax in Appendix A.3 and specifically in Proposition A.3.

where

$$\Delta \log M < 0,\ \Delta \log L^p > 0,\ \Delta \log \int z^{\sigma-1} m(z)\ \mathrm{d}z > 0. \tag{18}$$

There exists a unique $q^\star > q^{CES}$ (which generally depends change in fixed cost $\Delta \log f^c$) for which $\Delta \log Y = 0$. Furthermore,

$$\Delta \log Y < 0 \iff q > q^\star. \tag{19}$$

General equilibrium introduces an additional effect of recessions: the *labor-saving effect*. As the long-run economy features fewer operating firms, fewer labor resources are used for fixed costs as opposed to production. As a consequence, the income obtained by the household, both in terms of labor payments and profits, increases. To see this, note that the fixed supply of labor $\bar{L}$ is used in production L^p and for fixed costs payments Mf^c, since, by definition, there is no entry ins steady-state. Combining labor market clearing and the firms' pricing policy we note that nominal GDP $PY = \frac{\sigma}{\sigma-1}(\bar{L} - Mf^c)$. Hence, recessions that reduce the steady-state number of firms increase nominal GDP by reducing the amount of labor used for fixed costs payments and diverting it to production. Importantly, nominal GDP is also the total market size that firms consider upon entry: $PY = R$. As a consequence, the labor-saving effect induces additional entry after the recession, relative to our partial equilibrium framework. The direct consequence of the combination of these forces is an output gain, even in the CES case, which is highlighted in Remark 4.

Remark 4 (CES Aggregation in General Equilibrium)**.** *In the special case of CES aggregation $\left(q = q^{CES}\right)$, $\Delta \log Y > 0$.*

Based on the above intuition and remark, it is immediate that when $q > q^{CES}$, the economy values the loss of variety more than in the CES case. As a consequence, there exists a level of love-of-variety such that the household is indifferent between pre- and post-cycle outcomes as the three effects exactly cancel.

Proposition 4 establishes the properties of an economy experiencing a recession driven by some change in the fixed cost. In the remainder of the sections, we discuss many of the assumptions we have made so far and consider recessions of different intensities.

3.2 Discussion

We conclude this section with a brief discussion of the most salient model primitives and their interpretations. We provide a formal treatment of these extensions in Appendix A.

Aggregation and Love-of-Variety We have derived our results under a specific functional form assumption for the aggregator in eq. (1). This function has three key properties relative to a standard CES aggregator. First, there is a distinct parameter governing the love-for-variety effect, independently of the elasticity of substitution. Second, like the elasticity of substitution, love-of-variety is constant. Third, the variety effect constitutes an externality from the perspective of individual firms. All these features are important. In particular, the latter is key for our normative analysis, later in the paper. The first two are extremely convenient to obtain our transparent, closed-form characterization of our economy. However, the economic insight we put forth is much more general than our aggregator: recessions trigger a loss of variety and an improvement in average productivity; their welfare effects are governed by LoV and the price elasticity, respectively. LoV depends on the curvature of the utility function, while the price elasticity is the curvature of the marginal utility (demand). Matsuyama and Ushchev (2023) show that, in general, these two are functions of the number of available varieties. CES represents a special case in the class of Homothetic Single Aggregators in which these functions collapse to two scalars.

A second important assumption is that while firms are heterogeneous in their productivity, they are homogeneous in their contribution to the variety externality. We relax this assumption in Appendix A.4. We characterize the behavior of an economy where firms, upon entry, draw two distinctive features: their productivity z and their contribution to the variety externality ξ. These can be drawn from a joint distribution with arbitrary correlation. We show that our baseline economy is a special case of this more general setting where the z and ξ draws are independent. Away from this special case, we show in Proposition A.4 that the intensity of the correlation between productivity and externality contributions determines the long-run effect of recessions. Intuitively, if productivity and the externality contribution are positively correlated, recessions induce a smaller variety loss than if they are independent: the same cleansing effects that improve average productivity can lessen the loss of perceived variety losses. If the correlation is very positive, the effective number of varieties could even increase, generating long-run welfare gains independently of LoV q. Conversely, if the correlation between idiosyncratic externality and productivity is negative, the long-run effects of recessions are more negative than under the independence case. In this scenario, the cleansing effect induces a further loss as it negatively selects firms in terms of their externality contributions.

Entry, Exit, and Selection In our model, the path dependence of steady states is driven by an asymmetry embedded in the process of entry and exit. Firms enter under the veil of ignorance and draw their productivity. Selection occurs upon the draw, when firms with productivity $z < \underline{z}$ exit. As a consequence, as the economy is recovering from a recession, firms populate the entire support of the distribution above $\underline{z}$. Exit instead occurs in a strongly selected way. At the onset of a recession, firms leave the economy from the bottom of the distribution ($z < \underline{z}_2$). In this sense, recessions generate *selected* exit followed by *unselected* entry during recoveries. This property follows from our assumption that firms do not know their productivity upon paying the entry cost.

This modeling assumption provides us with a sharp and analytically tractable result, which qualitatively carries through in less extreme versions of our model. Suppose, for example, that all firms have some uncertainty about their true productivity z. As long as firms that have already produced have less uncertainty than prospective entrepreneurs, our results qualitatively hold. These results would also arise in a context in which potential entrants know their productivity exactly but face uncertainty about the fixed cost.

A related point regards our assumptions on exit. We have modeled exit as a forced event, triggered by negative current profits $\pi(z, m) < 0$. This assumption can be interpreted as firms defaulting due to working capital constraints and poorly developed financial markets. Suppose otherwise that firms could borrow against future profits. Then, they would make exit decisions by comparing the net present value of fixed cost $f^c/(1-\beta)$ to the NPV of gross profits $\widetilde{\pi}/(1-\beta)$. Under this reading, firm exit is not driven by a constraint, but by a rentability consideration. Due to the unanticipated nature of the shocks we consider, this formulation is equivalent to our static exit choice, as firm owners do not expect parameter changes. In Appendix A.5, we relax the assumption that firm owners do not expect any future changes in fixed cost, allowing them to forecast the reversion of fixed cost to pre-cycle levels. This extension is consistent with our propositions and does not alter results beyond a change in notation.

An additional important assumption in our framework is the absence of exit in the steady state. This is often generated by either idiosyncratic fluctuations in productivity (Hopenhayn, 1992) or some exogenous exit rate (Melitz, 2003). In either of those scenarios, the steady-state is not history dependent. As a consequence, all our results apply along the transition. We return to this point in the quantitative version of our model.

Aggregate Dynamics Proposition 1 provides a characterization comparing the economy before the shock to its long-run post-shock equilibrium. This allows us to isolate the Schumpeterian argument that cleansing recessions are beneficial in the long run. It also conveniently allows us to sidestep the full dynamic behavior of the economy along the transition. We provide an analytical characterization of the transition dynamics in Appendix A.6 under the assumption that the entrant productivity distribution is Pareto. In Propositions A.5 and A.6, we show that when fixed costs revert at a constant rate, the transition features a constant flow of entrants. The cutoff smoothly converges to its long-run level from above, while the number of firms converges from below. We show that fixed cost increases that slowly subside induce stronger selection effects as shown in Figure A.1.

Idiosyncratic Fluctuations Since our focus is on the consequences of aggregate shocks, we have considered an economy where firms are not subject to idiosyncratic fluctuations á la Hopenhayn (1992). Our model can be readily extended to account for idiosyncratic movements in productivity. We extend our results to such an economy in Appendix A.7.

Varieties and the Number of Firms In our model, varieties and firms are, by assumption, the same thing. However, our results readily generalize to economies in which firms produce multiple products. We assume that varieties produced by the same firm are more substitutable than varieties across firms, and we can then distinguish between LoV for products within and across firms. We analyse this extension and how our main results change in Appendix A.8. Our main conclusions continue to hold.

Fixed Costs Units In the model discussed in this section, we maintained that the unit of fixed and entry costs is labor. This provides a natural benchmark based on the Dixit and Stiglitz (1977); Dhingra and Morrow (2019) efficiency results, to which we return in our policy discussion. An alternative assumption is that these costs are in units of the final output good. This has important implications as entry then generates additional external effects (see Barro and Sala-i Martin, 2004). We consider this case in Appendix A.9 by extending our framework to one in which fixed costs are paid in units of a new good, made from a Cobb-Douglas aggregate of output and labor. This framework nests our baseline model and one of output good fixed costs as special cases. We show that our main results carry through.

3.3 Are larger crises more cleansing?

Until now, we have considered a fixed business cycle characterized by (f_l^c, f_h^c). A natural question is whether larger increases in the fixed cost induce larger cleansing effects and variety losses. To answer this, we start this analysis by noting that the effect of a marginally deeper recession can be summarized by the following elasticity:

$$\frac{\partial \log Y_3}{\partial \log f_h^c} = \frac{\partial \log M_3}{\partial \log f_h^c} \times \left[\frac{\partial \log Y_3^{CES}}{\partial \log M_3} + (q - q^{CES})\right], \tag{20}$$

where $Y_\tau^{CES} := L_\tau^p(\int z^{\sigma-1} m_\tau(z)\, \mathrm{d}z)^{1/(\sigma-1)}$ is the phase τ output in an economy with $q = q^{CES}$. We emphasize that eq. (20) is a local chain-rule identity for the *full* equilibrium response, *not* a partial derivative that holds the composition $m_\tau(\cdot)$ fixed. The sign of this elasticity determines whether a marginally bigger recession increases or decreases long-run output. This effect can be decomposed into two elements: i) the effect of larger fixed costs increases on a CES economy and ii) the additional variety effect. Both of these terms depend fundamentally on whether a larger fixed cost change increases or decreases the long-run number of available varieties. We study each element of eq. (20) in turn.

Effects of f_h^c on the mass of active firms M_3. We have argued above that the long-run number of firms necessarily declines after a crisis. This effect is driven by selection, as the average entrant is strictly more productive than the average exiter. This holds true for all recession intensities f_c^h and independently of LoV, q.

However, larger crises do not always imply marginally fewer firms in the long run: the sign of $\frac{\partial \log M_3}{\partial \log f_h^c}$ is ambiguous. The magnitude of the selection force that induces a decline in M_3 depends on the intensity of the recession f_c^h. To understand this, consider a very large recession. In such a case, the marginal exiting firm is very productive, in fact more productive than the expected entrant. As a consequence, even larger recessions induce less, not more selection. The opposite holds for very small recessions: a larger increase in f_h^c induces stronger selection effects. As selection effects drive the long-run number of firms M, we have that stronger recessions have ambiguous effects on M_3. We formalize this insight in Lemma 2.

Lemma 2 (Recessions Depth and Cleansing Effects)**.** *The long run number of firms active in the economy $M_3(f_h^c) \leq M_1$ attains a unique minimum M_3^{min} at some crisis level, $f_h^{c,*} \in (f_l^c, \infty)$. For each crisis driven by $f_h^c \in (f_l^c, f_h^{c,*})$ there exists some crisis $f_h^{c\prime} \in (f_h^{c,*}, \infty)$, such that $M_3(f_h^c) = M_3(f_h^{c\prime})$.*

The mechanism underlying this result is fully driven by the presence of incumbents. The marginal effect of the magnitude of the crisis on the long-run number of firms is given by

$$\begin{aligned} \left.\frac{\partial \log M_3}{\partial \log f_h^c}\right|_{f^c = f_h^c} &\propto \frac{\partial}{\partial f_h^c}\left[\underbrace{M_1 p_E(\underline{z}_2)}_{\text{Surviving Firms in Phase 2}} + \underbrace{E_3 p_E(\underline{z}_1)}_{\text{Entrants in Phase 3}}\right] \\ &= \left[\underbrace{M_1 \frac{\partial p_E(\underline{z}_2)}{\partial \underline{z}_2}}_{<0} + \underbrace{p_E(\underline{z}_1)\frac{\partial E_3}{\partial \underline{z}_2}}_{>0}\right] \cdot \underbrace{\frac{\partial \underline{z}_2}{\partial f_h^c}}_{>0}, \end{aligned} \tag{21}$$

where $p_E(\underline{z}) = \int_{\underline{z}}^{\infty} \mu^E(z)dz$. As f_h^c increases, the crisis cutoff $\underline{z}_2$ shifts up. At low levels of f_h^c, the first term in the bracket of eq. (21) dominates: marginal exiters are less productive than entrants, and there is a marginal decline in the number of firms. At high levels of f_h^c, the marginal exiter is more productive than the average entrant, and, therefore, any additional increase in the fixed cost is associated with marginally more firms in the long run.

Effect of M_3 on long-run output, Y_3. The elasticity of long-run output Y_3 with respect to M_3 decomposes into (i) the elasticity of Y_3^{CES}—output if the economy had LoV of level q^{CES}—and (ii) the elasticity of the variety effect in the q-economy relative to a CES economy. The latter is constant and its sign depends on whether $q \lesseqgtr 1/(\sigma - 1)$. The elasticity of Y^{CES} with respect to M_3 is always negative. A higher number of firms M_3 affects Y^{CES} in two different ways: i) more firms directly imply more fixed cost payments and, therefore, less labor used in production, and ii) a long-run equilibrium with more firms has undergone weaker selection along the transition. Both of these effects reduce Y_3^{CES}. However, note that $\frac{\partial \log Y_3^{CES}}{\partial \log M_3}$ is globally negative but not constant across different crisis intensities and independent of q. We can now return to the total effect of having more firms in the post-cycle steady state. On the one hand, additional firms generate one-to-one gains from varieties, provided that $q > q^{CES}$. On the other hand, additional firms generate more-than-linear reductions in the labor-saving and selection gains from the crisis, and thus in Y_3^{CES}. We conclude that reducing firm and hence product variety, M_3, always has a negative effect on long-run output if q is large and always has a positive effect if q is small.

In between these two extremes, reducing firm and hence product variety M_3 has a negative effect on long-run output for small values of M_3 (where the variety effect is stronger than cleansing and selection) and positive for large values of M_3. Next, we discuss this region of ambiguity and its implications for the effect of crises of varying magnitude, f_h^c, on total output.

Effect of f_h^c on Y_3. Taking stock, we know that as the crisis intensifies, the mass of varieties available in phase 3 first decreases then increases, and that this mass of varieties can have unambiguously positive, negative, or indeterminate effects on total output. We assemble these insights about the effect of f_h^c on Y_3 in Proposition 5:

Proposition 5 (Interaction of cycle depth and LoV)**.** *Index economies with otherwise equal parameters by their love-of-variety, q ('q-economies'). Then, there exists a unique, nonempty interval $(q_\circ, q^\circ)$ with $q_\circ > q^{CES}$ such that:*

1. $q \geq q^\circ \implies Y_3/Y_1 \leq 1$ *for all* f_h^c.
2. $q \leq q_\circ \implies Y_3/Y_1 \geq 1$ *for all* f_h^c.
3. *for all $q \in (q_\circ, q^\circ)$, larger crises can be welfare improving or welfare reducing depending on their intensity f_h^c.*

Proposition 5 provides three results. First, there exists a level of LoV $q_\circ$ such that any economy with $q \geq q^\circ$ faces GDP and welfare drops for any crisis intensity f_h^c. These economies value the presence of varieties so much that they experience long-run welfare losses, even for the smallest crises. Symmetrically, there is a subset of q-economies that do not value varieties as much and for which all recessions are long-run welfare-improving, independently of their intensity. Finally, the set of economies indexed by $q \in (q_\circ, q^\circ)$ is such that small and extremely large recessions can increase long-run output, while medium-sized recessions reduce it. This follows from the decreasing returns to labor-saving and selection effects as the number of varieties shrinks. We provide a graphical representation in Figure A.2 in Appendix A.10.

Given these ambiguous effects of recessions on long-run output and welfare, a natural question is whether a social planner would choose to distort the equilibrium allocation. In the next section, we focus on fixed cost cycles since they have the largest cleansing potential, and we characterize this policy problem.

4 Policy

In this section, we first study a social planner problem for economies with incumbents. The planner chooses the allocation of labor between entry costs, fixed costs, and each individual firm's production, controlling the entry mass $E_{\tau,t}^{SP}$ and the cutoff $\underline{z}_{\tau,t}^{SP}$ at time t and phase τ. We show that the government can use subsidies to achieve the first-best allocation with a single instrument: a tax/subsidy on f^e/f^c in eq. (7), financing the intervention using a lump-sum tax paid by the households. In the knife-edge case of CES, the market allocation is efficient. Next, we consider the optimal policy problem along the business cycle, as in Section 2.3. We find that the sign of the intervention is the same as the steady state one, but its optimal magnitude is larger: if the planner finds it optimal to subsidize firms in steady state, it should increase the size of the subsidies during recessions.

4.1 Planner Entry and Exit Choices

We consider a myopic planner maximizing current output. We show in Appendix B that the market's allocation of labor across firms is efficient. Hence, we allow the planner to choose directly the number of entrants E^{SP} and the cutoff productivity $\underline{z}^{SP}$. This is equivalent to choosing how much labor to allocate for production, entry costs, and fixed costs payments. Next, we decentralize allocation via a tax/subsidy. The planner's problem is

$$\max_{E_t^{SP},\underline{z}_t^{SP}} \quad Y_t^{SP} = (M_t^{SP})^{q-\frac{1}{\sigma-1}} L^{p,SP} \left[\int z^{\sigma-1} m_t^{SP}(z)\, \mathrm{d}z\right]^{\frac{1}{\sigma-1}} \qquad \text{s.t.} \tag{22}$$
$$\bar{L} \geq L_t^{p,SP} + f^e E_t^{SP} + f^c M_t^{SP},$$
$$m_t^{SP}(z) = (m_{t-1}(z) + E_t^{SP}\mu^E(z))\mathbb{I}_{\{z\geq \underline{z}_t^{SP}\}}(z),$$
$$\underline{z}_t^{SP}, E_t^{SP} \geq 0.$$

The myopic planner maximizes Y_t^{SP}, subject to the labor market clearing condition and the law of motion of the firms' productivity distribution. We solve the full dynamic problem in the quantitative version of the model. For the analytical characterization of the policy problem, we make the following assumptions:

Assumption 1. *We assume the following holds:*

a) The distribution of active firms is a truncation of the entrants' distribution: $m(z) \propto \mu^E(z)\mathbb{I}_{\{z\geq\underline{z}\}}$ for some z.

b) The expected relative market share of entrants decreases with higher truncations: $\mathbb{E}_{\mu^E}\left[(z/\underline{z})^{\sigma-1}|z\geq\underline{z}\right]$ decreases in $\underline{z}$.

Assumption 1a) amounts to an assumption on the history of the economy. It states that the distribution of active firms is proportional to the entrants' productivity distribution. This is the case whenever the economy has not undergone any recession yet, or it is in the middle of a fixed cost crisis. Assumption 1b) is an assumption on the productivity distribution of entrants. In particular, it posits that as we truncate the distribution from the left, the expected market share weakly decreases. This is satisfied by many commonly used distributions.[13] While we can relax these assumptions for specific results, we maintain Assumption 1 throughout this section to provide a consistent characterization.

Proposition 6 provides the characterization of the solution to the planner problem.

Proposition 6 (Social Planner Allocation)**.** *The following holds*

[13] Distributions satisfying the assumption are, for example, Pareto, Exponential, Weibull (for shape parameter $k > 1$), Burr, and Fréchet (for shape parameter $\xi \leq 1$). It can be interpreted as requiring that as the condition for survival becomes more stringent, the productivity of marginal exiter and new entrant become more similar on expectation. A sufficient condition for $\mathbb{E}_{\mu^E}\left[(z/\underline{z})^{\sigma-1}|z\geq\underline{z}\right]$ to be decreasing in the cutoff $\underline{z}$ is that the elasticity of the anti-cumulative corresponding to μ^E with respect to the truncation $\underline{z}$, $\varepsilon_{1-F}(\underline{z})$, is globally larger than -1, i.e. $\varepsilon_{1-F}(\underline{z}) \geq -1$, $\forall \underline{z} \geq 0$.

I) The planner solution coincides with the market allocation if and only if $q = q^{CES}$.

II) In a state of entry,

a) The optimal number of entrants E^{SP} is strictly increasing in q.

b) The presence of incumbents with higher average productivity than entrants decreases the socially optimal number of entrants and, therefore, the number of firms. However, the presence of incumbents does not affect the social planner's steady-state cutoff.

c) If there are no incumbents, the cutoff $\underline{z}^{SP}$ is decreasing in q.

III) The socially optimal allocation can be achieved in a decentralized equilibrium where the planner sets a subsidy/tax to the fixed cost θ^c such that firms pay $f^c(1-\theta^c)$, financed by a lump-sum tax on the household. The optimal subsidy/tax θ^c satisfies

$$1 - \theta^c(\underline{z}^{SP}) = \left[[q(\sigma-1)-1] \left(\mathbb{E}_{\mu^E}\left[\left(\frac{z}{\underline{z}^{SP}} \right)^{\sigma-1} \mid z \geq \underline{z}^{SP} \right] \right)^{-1} + 1 \right]^{-1}. \tag{23}$$

Where $\theta^c \gtreqless 0 \iff q \gtreqless 1/(\sigma-1)$.

The result in Proposition 6 extends previous insights from Spence (1976); Dixit and Stiglitz (1977); Mankiw and Whinston (1986); Parenti et al. (2017); Bilbiie et al. (2019); Dhingra and Morrow (2019) and Matsuyama and Ushchev (2020) to economies with firm heterogeneity and incumbents. When the planner is constrained by the presence of already existing incumbent firms, it optimally chooses the same cutoff and mass of entrants as in the market equilibrium if and only if the intensity of love-of-variety is that of CES preferences. If LoV is larger, there are inefficiently few firms operating in the market. These potential inefficiencies represent steady-state distortions. For any LoV, the optimal number of entrants is decreasing in the mass incumbent. In our setting, where incumbents are, on average, more productive than successful entrants, the planner optimally includes fewer firms the larger the mass of incumbents. Intuitively, the social planner internalizes the trade-off between allocating more labor to the most productive firms versus producing more varieties through a less productive entry margin. Without incumbents, LoV affects the cutoff in an intuitive way: a higher q leads the social planner to decrease $\underline{z}$ and avoid further exit.

Finally, Proposition 6.III) characterizes the optimal policy intervention. A subsidy/tax to fixed costs is sufficient to restore efficiency. Importantly, whether the planner wants to subsidize or tax the presence of firms in the economy depends exactly on whether q is larger or smaller than $1/(\sigma-1)$.

Next, we characterize the optimal policy intervention during business cycles.

4.2 Optimal Policy through the Cycle

We compare the planner solutions and market allocations subject to a cycle like in Section 2.3. Figure 4 illustrates the comparison. Suppose that $q > 1/(\sigma-1)$, then in the original allocation

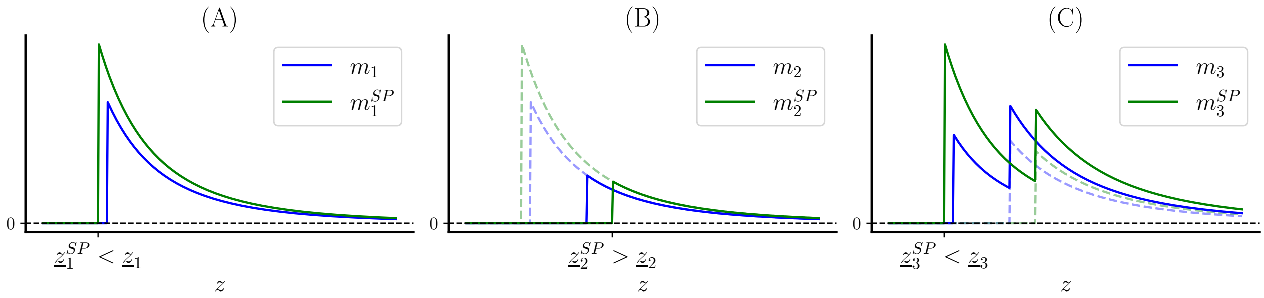

Figure 4: Evolution of m through the business cycle for the market and social planner allocations, assuming that $q > q^{CES}$.

there are too few firms and the planner induces additional entry by implementing the optimal policy from Proposition 6.III). Before the recession, the planner allocation features more firms, as shown in Panel (A). Suppose that the two economies experience an increase in the fixed cost. The planner's economy, by virtue of the larger measure of firms, is effectively hit harder by the rising fixed costs. As a consequence, the cutoff experiences a much larger rightward shift than in the laissez-faire allocation. Importantly, the planner finds it optimal to increase the subsidy during the crisis, which counteracts this force, leaving the realized cutoff to the right of the laissez-faire economy but not as far to the right, preventing the exit of some firms. When the rise in fixed costs is reabsorbed, firms re-populate the economy. Since the planner is instating the steady-state subsidy from Proposition 6, more firms enter, and the post-crisis distribution features more varieties than the market allocation would deliver. We summarize the optimal policy response in the three phases of the cycle.

Proposition 7 (Optimal Policy over the Cycle)**.** *The optimal fixed cost taxes/subsidies are counter-cyclical:* $|\theta_1^c| \leq |\theta_2^c|$ *and* $|\theta_3^c| \leq |\theta_2^c|$.

The social planner's optimal policy is to increase the magnitude of its steady-state interventions, determined by Proposition 6.III). If the economy features $q > 1/(\sigma - 1)$ then the planner's steady-state policy is a subsidy $\theta_1 > 0$, in which case during period 2 at the peak of the crisis, the planner optimally increases the subsidy to $\theta_2 > \theta_1$. The intuition is that, during the recession, firms' exit permanently lowers the number of available varieties. To avoid this long-run loss, the planner intervenes by softening the blow and avoiding the exit of some of the firms. As the recession reabsorbs, the planner reduces the subsidy to a lower level $\theta_3 < \theta_2$. Conversely, if $q < 1/(\sigma - 1)$, the optimal steady-state policy is a tax $\theta_1 < 0$ and the planner increases this tax during the recession. In this scenario, the planner takes advantage of the crisis as a cleansing moment and uses it to *pick winners* in the long-run steady state. We conclude that the social planner implements an optimal cyclical policy, which can be inferred from their steady-state behavior.

Given the importance of *love-for-variety* q and the price elasticity of demand σ to determine the optimal intervention, in the next section, we provide a new approach to identify them separately.

5 Estimation

In order to quantify the forces of interest, we set out to estimate our two key parameters: q and σ, governing love-for-variety and substitutability across varieties. We estimate q and σ in a cross-industry version of our model for all the countries available in the World Input-Output Database (Timmer et al., 2015), henceforth WIOD, as well as in the Spanish administrative firm-level data (SABI). Since we consider a large number of different industries, $i \in \mathbf{I}$, over time, $t \in \mathbf{T}$, and across countries, $c \in \mathbf{C}$, we study the partial equilibrium framework of Section 2.3. We discuss the identification of q and σ in turn.

5.1 Identification of q

The estimation of love-for-variety has been extremely elusive for decades. The main difficulty is that it requires exogenous variation in the number of available varieties to estimate its effect on output. This, naturally, requires taking a stance on what a variety is in the data. Our starting point to estimate q is different: we use a structural relation of the model, linking changes in expenditure to changes in output through q. We start by taking logs of the aggregate production function in eq. (8), and differentiating with respect to $\log \mathcal{I}$ to obtain

$$\frac{\partial \log Y}{\partial \log \mathcal{I}} = q \frac{\partial \log M}{\partial \log \mathcal{I}} + 1 + \frac{\partial \log \bar{z}}{\partial \log \mathcal{I}}. \tag{24}$$

A change in nominal expenditure of downstream customers on the industry, $\mathcal{I}$, induces a 1-for-1 direct increase in output as well as two indirect effects. First, higher expenditure induces entry, which in turn increases M. This effect generates a change in output that depends only on q. Second, by inducing entry, higher $\mathcal{I}$ changes the average productivity of firms, $\log \bar{z}$, provided entrants/exiters do not have the same average productivity as incumbents. Since in equilibrium production labor L^p is proportional to $\mathcal{I}$, the same equations and all following results hold for changes in L^p, too. Next, we note that the economy does not respond symmetrically to expansions and contractions. In an expansion, the average productivity of firms in the economy changes if entrants are more or less productive than incumbents on average. Differently, a reduction in $\mathcal{I}$ always increases average productivity, since exit occurs from the bottom of the distribution. The following Lemma characterizes the elasticities given *small increase in* $\mathcal{I}$.

Lemma 3 (Elasticities In Output Equation)**.** *Suppose, entrants and incumbents are distributed according to* $m^E(\cdot) \propto \mu^E(\cdot \mid z \geq \underline{z})$ *and* $m^I(\cdot) = E^I \mu^E(\cdot \mid z \geq z^*), E^I \geq 0$, *respectively.*[14] *Then, the right-derivatives* $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}$ *and* $\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}}$ *are given by:*

$$\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}} = \frac{1}{\sigma - 1}\left(1 - \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}\right), \quad \textit{and} \qquad \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} = \frac{E_{|E^I=0} p_0(\underline{z})}{E_{|E^I=0} p_0(\underline{z}) + E^I\, k(\underline{z}, z^*)}, \tag{25}$$

[14] Here, E^I is the number of 'historical' entrants, i.e., the mass of incumbent firms that have tried, and of which $E^I p_E(z^*)$ have succeeded to enter the market.

where $E_{|E^I=0}$ is the number of entrants if there are no incumbents, and k is a function satisfying $k(\underline{z}, z^*) \lesseqgtr 0$ if $\underline{z} \lesseqgtr z^*$.

For identification, we make Assumption 2, which states that the latent entrant distribution and the incumbent distribution have the same cutoff. This has three interpretations. Either (i) the industry has not undergone a sequence of selection-inducing fixed-cost recessions yet. Or (ii) the industry is in a long-run equilibrium with exogenous death shocks (cf. Proposition A.8), such that the cleansing effects since the last recession have fully subsided. Or, finally, (iii) entrants 'learn' from incumbents, such that their technologies, conditional on clearing the cutoff, $\underline{z}$, are no worse than those of existing businesses.

Assumption 2 (Entrant and Incumbent Distribution)**.** $\underline{z} = z^*$.

Under Assumption 2, entry induced by shocks such as income expansions that leave f^c/f^e and hence the cutoff unchanged do not affect average productivity, $\bar{z}$. As a consequence, we have the following identification result.

Proposition 8 (Identification of q)**.** *Under Assumption 2,*

$$\frac{\partial_+ \log Y}{\partial_+ \log X} = 1 + q, \quad X \in \{\mathcal{I}, L^p\}, \tag{26}$$

and in the regression equation

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \gamma \Delta \log \bar{z}_{i,t} + \epsilon_{i,t}, \quad X_{i,t} \in \{\mathcal{I}_{i,t}, L^p_{i,t}\}, \tag{27}$$

we have $\Delta \log X_{i,t} > 0 \implies \Delta \log \bar{z}_{i,t} = 0$ and $\beta = 1 + q$.

This result allows us to identify q as the determinant of the elasticity of output to income changes. Underlying this result is that, by the free entry condition, an increase in income $\mathcal{I}$ increases the number of firms proportionally. The change in M induces a direct change in output with elasticity q. In this sense, we estimate q as governing the intensity of external returns to scale over and above the technological returns, which in our model are equal to 1 by the constant returns to scale assumption. Importantly, this only holds for positive changes in income and demanded labor since any negative change necessarily induces exit from the bottom of the distribution and, therefore, a response in $\log \bar{z}$. We conclude that we can estimate love-of-variety q provided some exogenous increase in employment L^p or expenditure $\mathcal{I}$ from

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \epsilon_{i,t}, \quad X_{i,t} \in \{\mathcal{I}_{i,t}, L^p_{i,t}\}. \tag{28}$$

Proposition 8 enables us to estimate eq. (28) with an instrument inducing *positive variation* in L^p or $\mathcal{I}$, while disregarding the average productivity term. We use this identification strategy as our benchmark and discuss possible threats to identification in the next section. In particular, how eq. (27) directly extends to a setting with multiple factor inputs, non-constant returns to scale, and measurement error. Additionally, we discuss how failure of Assumption 2 still allows us to identify the sign of $q - q^{CES}$, given that we can identify $q^{CES} = \frac{1}{\sigma-1}$, or a lower bound thereof.

5.2 Discussion of Identification

Multiple Factor Inputs Part of our identification result in Proposition 8 relies on the assumption that labor is the only factor of production and that it enters the production function linearly. This how we obtain the "1" in our estimated coefficient of $\beta = 1 + q$. Clearly, these assumptions are counterfactuals in empirical applications. Our identification strategy readily extends to settings in which firms have multiple inputs combined under constant returns to scale (CRS). The "1" we identify is our regression should be interpreted as the returns to the optimal input bundle (in our model, trivially made only of labor). Importantly, in the absence of relative input price changes, homogeneity of the production function implies that factor demands are proportional to one another. As a consequence, estimating eq. (28) on a single input (labor) does not affect our finding. Hence, with multiple inputs in CRS, we would still obtain $\beta = 1+q$ from estimating eq. (28). If the production function had non-constant returns to scale, we would have an upward bias in our estimated q if the true data-generating process features increasing returns, and a downward bias if it features decreasing returns to scale. We summarize both these results in Remark 5.

Remark 5 (Identification for Homogeneous Production Functions)**.** *Proposition 8 holds more generally for any production function that is homogeneous of degree 1. If the production function is instead homogeneous of degree χ, the right derivative with respect to some input X estimates*

$$\frac{\partial_+ \log Y}{\partial_+ \log X} = \chi + q.$$

Recent evidence estimating returns to scale points to small deviations from CRS, with returns to scale of at most 1.05 (see for example Chiavari, 2024). This upper bound implies a maximal upward bias of just 0.05 in our estimate of q, leaving our findings unaltered.

Mismeasurement of Price Indices An important requirement in our estimation approach is the observation of output, rather than sales. In our empirical analysis, we use deflated sales to obtain output. This relies on the correctness of the industry deflators as composites of the standard CES-implied price index P^{CES} and the variety effect $M^{\frac{1}{\sigma-1}-q}$. A natural question when taking our approach to the data is whether the WIOD deflators incorporate the variety effect—and, accordingly, whether and under what assumptions q is identified, and how large the bias can be when one recovers q as per Proposition 8, i.e. $\widehat{q} = \widehat{\beta} - 1$. We consider this in Remark 6. To fully characterize this problem, which is not always defined when deflators are incorrectly measured, we consider an economy where fixed and entry costs are produced by a Cobb-Douglas bundle of output (share α) and labor (share $1 - \alpha$) as described in Appendix

A.9.[15] Denote with $\sim$ objects measured or estimated using the incorrect deflators, which do not account for the variety effect. For example, call $\widetilde{Y}$, sales deflated by P^{CES} rather than P.

Remark 6 (Mismeasurement in Price Indices)**.** *Under Assumption 2 and positive demand shocks, the regressions*

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \epsilon_{i,t}, \qquad \Delta \log \widetilde{Y}_{i,t} = \widetilde{\beta} \Delta \log X_{i,t} + \epsilon_{i,t}, \qquad X_{i,t} \in \{\mathcal{I}_{i,t}, L^p_{i,t}\} \tag{29}$$

identify

$$\beta \;=\; 1 - \frac{q}{\alpha q - 1}, \qquad \widetilde{\beta} \;=\; 1 - \frac{1}{(\sigma-1)(\alpha q - 1)}. \tag{30}$$

Hence, given an estimate β or $\widetilde{\beta}$, and σ, the implied love-of-variety parameter q is

$$q(\alpha) = \frac{1-\beta}{\alpha(1-\beta)-1}, \qquad \widetilde{q}(\alpha) = \frac{1}{\alpha}\left[1 + \frac{1}{(\sigma-1)(1-\widetilde{\beta})}\right]. \tag{31}$$

Taken together, these relationships deliver two results that allow us to bound our estimate against mismeasurement of price indices.

(i) When empirical deflators account for varieties, q is identified for any $\alpha \in [0,1]$. Furthermore, the bias is given by $-\frac{\alpha(1-\beta)^2}{\alpha(1-\beta)-1}$, for $\alpha(1-\beta) \neq 1$.

(ii) When deflators do not account for varieties, q is identified for any $\alpha \in (0,1]$ conditional on σ. Furthermore, the bias is given by $\frac{\alpha(\sigma-1)(\widetilde{\beta}-1)^2-(\sigma-1)(\widetilde{\beta}-1)+1}{\alpha(\sigma-1)(\widetilde{\beta}-1)}$, for $\alpha > 0$ and $\widetilde{\beta} \neq 1$.

In summary, identification holds for any α when deflators are correctly reported, and for any $\alpha > 0$ (given σ) when deflators fail to incorporate the variety effect. If one recovers q using the result in Proposition 8, in anticipation of the empirical results, the bias is positive but small in the correctly-measured deflator case, while in the mismeasured case the bias is negative for our estimate of β and σ. Our findings remain unaltered when taking into account the bias under both deflators. We provide these bounds together with our main estimates in Section 5.6.

Estimates without Assumption 2 Our identification strategy requires that there is at most negligible selection in incumbents. However, even if Assumption 2 fails entirely and $\underline{z} < z^*$ holds, we are still able to identify the sign of $q - q^{CES}$ and determine whether recessions (in partial equilibrium) are welfare-improving. This is formalized in the following proposition.

[15] We adopt this formulation because when deflators are incorrectly measured, the limit of our estimator is not informative on q, and, therefore, does not have a bounded bias. Using this more general model allows us to characterize the limit behavior of the bias and therefore find its sign.

Proposition 9 (Identification of $\text{sgn}(q - q^{CES})$). *Suppose that* $q^{CES} = \frac{1}{\sigma-1}$ *is identified. Let* β *be the regression coefficient on* $\Delta \log \mathcal{I}_{i,t}$ *in eq. (28). Then, the bias of* $\widehat{q} = \beta - 1$ *is given by*

$$bias_{+} = \widehat{q} - q = \left(\frac{1}{\sigma - 1} - q\right)\left(1 - \frac{\partial_{+} \log M}{\partial_{+} \log \mathcal{I}}\right), \tag{32}$$

assuming an expansion in $\mathcal{I}$*. Furthermore, for* $\underline{z} < z^{*}$,

$$\text{sgn}\left(\widehat{q} - \frac{1}{\sigma - 1}\right) = \text{sgn}\left(q - \frac{1}{\sigma - 1}\right), \tag{33}$$

Which implies that even without Assumption 2, we can identify the sign of $q - q^{CES}$.

Firms, Establishments, and Varieties As a final remark, note that our approach circumvents a fundamental problem in mapping the model to the data: the definition of a variety. In principle, if we assumed that establishments/firms only produced a single variety, we could use the number of firms or establishments as a direct measure of M. This mapping is fragile as establishments/firms may produce multiple products (as we highlight in the extension section), or new establishments of firms, such as factories, stores, or offices, may indicate larger quantities of existing rather than new product varieties. In such a case, we would need to observe both the number of firms and the number of products per firm. Our empirical strategy sidesteps this issue since it relies on the observation that q governs the intensity of external aggregate returns to scale. Our approach is vulnerable to the criticism that identification relies on the absence of other sources of spillovers. Conversely, alternative approaches to the estimation of love-for-variety require a precise definition of variety, for example Baqaee et al. (2023). We return in the results section to the comparison of our findings.

5.3 Data

We use two main data sources: the World Input-Output Database (WIOD) and the Spanish administrative balance sheet data (SABI). We use the former to estimate our key parameter of interest and the latter for validation and calibration.

Input-Output Data The primary data source for the identification of q is the World Input-Output Database (WIOD) 2016 release, see Timmer et al. (2015). It contains the Input-Output structure of sector-to-sector flows for $C = 44$ countries from 2000 to 2014 yearly. The data is available at the 2-digit ISIC rev-4 level. The number of sectors in WIOD is $I = 56$, which amounts to 6,071,296 industry-to-industry flows and 108,416 industry-to-country flows for every year in the sample.

The World Input-Output Table is an $(I \times C)$ by $(I \times C)$ matrix whose entries. Each element Z_{cd}^{ij} denotes the sales of industry i in country c to industry j in country d for intermediate input use. Additionally, the data includes an $(I \times C)$ by C matrix of final use. Each element F_{cd}^{i} is the value of sales of industry i in country c sold to and consumed by households, government, and non-profit organizations in destination d. Denote $F_c^i = \sum_d F_{cd}^i$ the value of output of sector i in country c consumed in any country in the world. The total value of sales is $S_c^i = F_c^i + \sum_d \sum_j Z_{cd}^{ij}$.

WIOD also includes industry-level deflators. Importantly, the data is also provided at previous-year prices, which allows us to concatenate the data to obtain changes in output from changes in sales at different prices, as we explain in Appendix C.1.

We complement the I-O table with the data in the Socio-Economic Accounts, which include information on input usage for all sector-country-year tuples. In particular, we make use of the information on labor used in production by these industries.

Spanish Administrative Data (SABI) The Sistema de Análisis de Balances Ibéricos (SABI) is a firm-level database maintained by Bureau van Dijk and Informa D&B. It contains standardized financial statements and company information for more than two million Spanish firms, drawing on official sources such as the Boletín Oficial del Registro Mercantil (BORME), the Mercantile Registry, and stock exchange filings. The database provides historical annual accounts, including balance sheets and income statements. Our sample covers the period 2000–2018 at an annual frequency.

5.4 Implementation - Instrumental Variable

For each of the WIOD industries, we would like a plausibly exogenous shift in expenditure. We build this as in Ferrari (2024) through a shift-share, aggregating destination-specific shifters through destination shares. Formally, an industry i in country c at time t is assigned a change in demand equal to

$$\sum_j \xi_{cd}^i \Delta \log F_{dt}, \tag{34}$$

where ξ_{cd}^i represents the fraction of the value of output of industry i in country c consumed directly or indirectly in destination d in the first sample period and $\Delta \log F_{dt}$ is the growth rate of final consumption expenditure in destination d at time t. The exposure share ξ_{cd}^i, assumed to be time-invariant, includes direct sales from the industry to consumers in d and output sold to other industries, which eventually sell to consumers in d.

The input-output structure of the data allows a full account of these indirect linkages when analyzing sales composition. Formally, define the share of sales of industry i in country c that is consumed by destination d as

$$\xi_{cd}^i = \frac{F_{cd}^i + \sum_j \sum_e a_{ce}^{ij} F_{ed}^j + \sum_j \sum_e \sum_k \sum_f a_{ce}^{ij} a_{ef}^{jk} F_{fd}^k + \ldots}{S_c^i}, \tag{35}$$

where a_{cd}^{ij} is dollar amount of output of sector i from country c needed to produce one dollar of output of sector j in destination d, defined as $a_{cd}^{ij} = Z_{cd}^{ij}/S_d^j$. The first term in the numerator represents sales from sector i in country c directly consumed by d; the second term accounts for the fraction of sales of sector i in c sold to any producer j in country e that uses c, i as input and then sells to destination d for consumption. The same logic applies to higher-order terms. By definition $\sum_d \xi^i_{cd} = 1$.

For the shifters, we estimate the common component of all industries selling to a given destination d at time t. For each industry, we iteratively exclude flows from the same country and in the same sector

$$\Delta \log F^j_{edt} = \eta_{dt}(c, i) + \nu^i_{edt} \quad e \neq c, j \neq i. \tag{36}$$

This boils down to identifying our effect of interest through changes in foreign demand on other products; we refer to Ferrari (2024) for an extended discussion of the identification and briefly discuss the key intuition. The idea behind our instrumental variable approach is that each industry is small relative to each destination country. As a consequence, variations in aggregate consumption in destinations are quasi-randomly assigned to each producing industry through pre-existing network linkages. The exclusions in eq. (36) state that we only use variation from different products and different countries to reduce concerns of reverse causation. Formally, our instrument is given by

$$\eta^i_{ct} = \sum_d \xi^i_{cd} \widehat{\eta}_{dt}(c, i). \tag{37}$$

Following Proposition 8, we estimate

$$\Delta \log Y^i_{ct} = \beta \Delta \log \widehat{X}^i_{ct} + \epsilon^{+i}_{ct}, \tag{38}$$

Where $\Delta \log Y^i_{ct}$ is the growth rate of output sold by industry i in country c at time t, which we compute as described in Appendix C.1. $\Delta \log \widehat{X}^i_{ct}$ is the instrumented change in either expenditure $\mathcal{I}$ or production labor L^p.

5.5 Identification of σ

In our model, σ can be identified directly from the profit share since $\pi/R = 1/\sigma$. However, the WIOD data is built such that the sum of material, labor, and capital expenditure is equal to gross output. This implies lumping together economic profits with the cost of capital. For the purpose of identifying σ, we make the conservative assumption that all capital expenses are, in fact, profits. Hence, we compute this profit rate as gross output net of worker compensation and intermediate input expenditures divided by gross output: $1/\sigma = (GO - COMP - II)/GO$ for all industry-country-year triplets. Clearly, this implies that we are overstating the profit share in our measurement. As a consequence, our estimated σ is a lower bound for the true elasticity of substitution and, therefore, the implied q^{CES} is larger than the true q^{CES}. Alongside the WIOD-based measurement, we compute σ using the universe of firms in the Spanish SABI database. We apply the same conservative treatment of capital costs as profits, which similarly leads to an upward-biased value of q^{CES}.

5.6 Results

Estimates of q^{CES} We start with a description of our estimated σ. Based on the WIOD data, we obtain the distribution of profit rates across industries summarized in Table 1. The profit rate ranges between 0% and 69.9% with a median of 17.2%. The corresponding σ distribution is between 1.4 and 10, with a median of 5.8, in line with the estimates in the literature (see Anderson and Van Wincoop, 2004, for a review). We are interested in the q^{CES} associated with this distribution of elasticities. We find q^{CES} to be between 0.11 and 2.32, with a median of 0.21. In the SABI data, we find a median profit rate of 18.6%, as shown in the right panel of Table 1. The associated q^{CES} of 0.23 is our chosen measure of q^{CES} for the quantitative application in Section 6, providing a more conservative benchmark than the WIOD median.

Table 1: Distribution of Profit Rate, σ, and q^{CES}

	WIOD			**Spanish administrative data (SABI)**		
Percentiles	**Profit Rate**	σ	$\mathbf{q^{CES}}$	**Profit Rate**	σ	$\mathbf{q^{CES}}$
1	0.0%	.	.	-12.1%	.	.
5	10.0%	10.0	0.11	3.6%	27.6	0.04
10	12.2%	8.2	0.14	6.8%	14.6	0.07
25	13.0%	7.7	0.15	11.7%	8.5	0.13
50	17.2%	5.8	0.21	18.6%	5.4	0.23
75	23.8%	4.2	0.31	28.4%	3.5	0.40
90	31.3%	3.2	0.46	40.8%	2.5	0.69
95	34.4%	2.9	0.52	49.7%	2.0	0.99
99	69.9%	1.4	2.32	65.3%	1.5	1.88

Notes: The empirical distribution pertains to the profit rate; the values for σ and q^{CES} are obtained from the profit rates through the theoretical relationships set out in the text. For the SABI panel, we omit the "Percentiles" column and align rows to the left panel's percentiles. For SABI, we keep only firms with employees ≥ 5, as profit rates of very small firms are outlier-prone. The definition of profits includes capital expenses, consistently with the WIOD definition. Results are very similar when including all firms (median $q^{CES} = 0.24$).

Estimates of q We estimate love-for-variety q by means of eq. (38). Since the procedure holds for changes in both expenditure and labor usage, we report results for both in Table 2. As the positive demand shocks could induce firm entry with a delay, we use both contemporaneous and lagged shocks as instruments.[16] The first and second columns in each panel show the first- and second-stage estimation results, respectively. In all our estimations, we find a significant first stage and βs larger than 1. Recall that $q = \beta - 1$, which implies that all our estimates of q are strictly positive. Depending on the specification, we find estimates of q between .512 and .634. For comparison, using a different identification and firm-level data, Baqaee et al. (2023) estimate a love-for-variety elasticity of .3. We interpret our larger estimate as a sign of even stronger aggregate increasing returns than implied by individual firms' production functions.

Panel (C) in Table 2 reports the results of an empirical model treating eq. (38) for labor and

[16] In Appendix C.2, we show that our results are robust to this choice. We also show that the findings hold when weighting observations by industry size.

income as not independent. We estimate a 3SLS model explicitly accounting for the correlation between the error terms across the system of equations. Additionally, we impose common coefficients, as implied by our model. We choose the estimate of q from panel (C), 0.568, as our q for the quantitative application in Section 6.

Table 2: Estimation Results

	(A)		(B)		(C)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^i_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log L^{p,i}_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log Y^i_{c,t}$
$\eta^{+,i}_{c,t}$	0.0854***		0.103***		
	(0.0215)		(0.0204)		
$\eta^{+,i}_{c,t-1}$	0.0941***		0.0646***		
	(0.0231)		(0.0204)		
$\Delta \log \mathcal{I}^i_{c,t}$		1.512***			1.568***
		(0.191)			(0.151)
$\Delta \log L^{p,i}_{c,t}$				1.634***	1.568***
				(0.245)	(0.151)
$P(q \leq q^{CES})$		.055		.040	.008
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	10067	10067	10083	10083	10067
F-Stat	22	63	23	45	
R^2	0.0315	0.0864	0.0413	0.0881	
F-Stat eq.(1)					2843
F-Stat eq.(2)					131
R^2 eq.(1)					0.0866
R^2 eq.(2)					0.0877

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

To check the robustness to mismeasurement in deflators, we can invoke Remark 6. If deflators correctly incorporate the variety effect, the implied q lies between 0.362 and 0.568. If deflators instead neglect the variety effect, then, given any non-zero output share in fixed cost production $\underline{\alpha}$, the implied q lies between 0.595 and $0.595/\underline{\alpha}$. Note that, if statistical agencies correctly report deflators, our lower-bound remains larger than q^{CES}. If instead deflators are mismeasured, our estimate is a lower bound for the true value of love-of-variety.

In Figure 5, we provide a visual representation of our results. We plot our chosen estimate of q from panel (C) against the distribution of q^{CES} at the industry level. Our estimate implies that only 2 industries out of 56 in our sample have $q^{CES} > q$, and for the vast majority of industries, q is substantially larger than q^{CES}. Furthermore, the median q^{CES}, which we use as our benchmark, lies below the lower bound of the two-sided 95% confidence interval for our estimated q. Therefore, when conducting a one-sided t-test, we can reject the hypothesis that $q \leq q^{CES}$ at the 5% significance level as per Table 2. This result suggests that recessions have a significant potential to reduce long-run welfare as agents are sensitive to the loss of varieties.

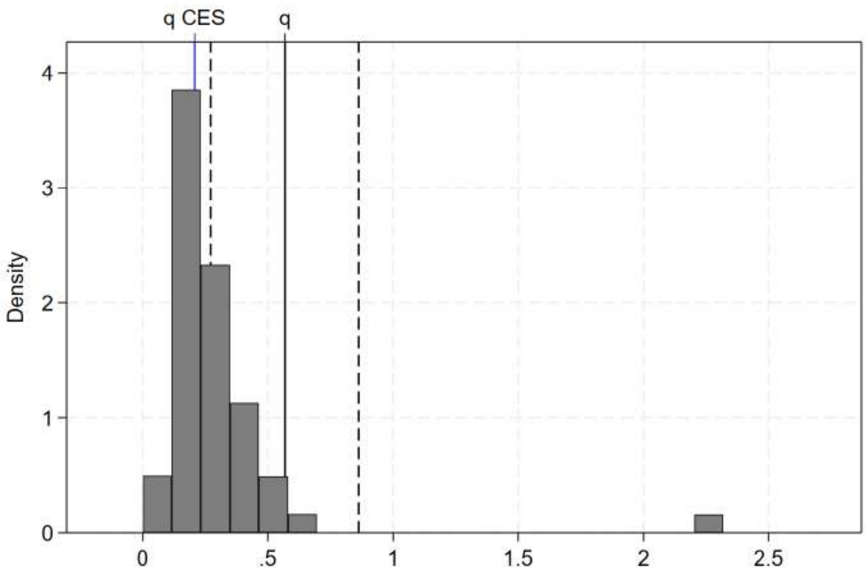

Figure 5: Estimated q and q^{CES}. The plots the distribution of estimates of $1/(\sigma - 1)$ by industry, highlighting the median, which is our preferred point estimate for q^{CES}. Additionally, it shows our estimate of a global q, with 95% confidence bands.

6 Quantitative Model

We conclude by considering a quantitative version of our model to study fixed-cost cycles and the optimal policy response. We simulate firm exit and entry in a mean-reverting fixed-cost cycle and characterize output losses in terms of log deviations from the steady state along the transition.

6.1 Set-Up and Calibration

We consider a strict generalization of our analytical model. In particular, we allow firms to know the full path of the fixed cost after the initial shock. Firms discount the path of fixed cost with factor $\beta \in (0, 1)$. They forecast

$$\pi(z, \mu_t) = \frac{\widetilde{\pi}(z, \mu_t)}{1 - \beta} - \sum_{0 \leq \tau < \infty} \beta^\tau \bar{f}^c_{t+\tau}, \tag{39}$$

where $\widetilde{\pi}$ are profits gross of fixed cost payments. Fixed cost after potential government subsidies are given by $\bar{f}^c_t$. Firms estimate the NPV of expected future profits by anticipating the transition of fixed costs after subsidies. The net present value of fixed cost, $NPV(\bar{\mathbf{f^c}})$, determines the cutoff and enters the eqs. (4), (5) in place of f^c. We also include random exit shocks in the analysis, i.e., at the beginning of each period, a fraction $\delta \in (0,1)$ of firms exit, which yields a unique steady-state of the economy in the long run. The factor $(1-\delta)$ is subsumed into the firm discount factor. We set $\delta = 3.36\%$, which is the average exit rate of all active Spanish firms, as covered by SABI, in the period preceding the Global Financial Crisis (GFC). As our sample begins in 2000, the average is calculated from 2000 to 2006. We assume that the entrant productivity distribution, μ^E, is Pareto, and parameterized by location $z_{min} = 1$. For the shape parameter, we follow Gabaix and Ibragimov (2011) to estimate the shape parameter of the pre-GFC revenue distribution in SABI. Denoting h the shape parameter of the firm size distribution, in our model, the productivity distribution has a shape parameter of $h(\sigma-1)$. The elasticity of substitution, σ, and LoV, q, were estimated in Section 5. The log-deviations of model quantities are invariant to the scale of $f^e > 0$, which therefore does not need to be calibrated. For the same reason, we normalize $\bar{L} = f_0^c = 1$, where f_0^c refers to the steady state fixed cost.

The social planner solves the full dynamic Ramsey problem, accounting for the future effects of policy interventions in the current period. The planner maximizes welfare with some discount factor, β^* and a social welfare function $U(\mathbf{Y}) = \sum_t (\beta^*)^t \log(Y_t)$. The planner's discount factor is linked to the firms' through $\beta^* = \beta/(1-\delta)$. Namely, $\beta^\star$ is the social discount factor, and firms discount faster due to the probability of exit.[17] We allow the planner to choose two policies $\theta_{ss}, \theta_{cyc}$ to interact with the steady state and cycle fixed cost, respectively. Suppose that the economy is in its steady state in period 0, and a crisis occurs in period 1, such that $f_t^c > f_0^c$, $\forall t \geq 1$. Given the policy levers θ_{ss} and θ_{cyc}, the fixed cost paid by firms is

$$\bar{f}_t^c = (1-\theta_{ss})[f_0^c + (1-\theta_{cyc})(f_t^c - f_0^c)]. \tag{40}$$

Hence, θ_{ss} is a subsidy (or tax) on the overall fixed cost f_0^c. θ_{cyc} is a subsidy/tax on the temporary change $f_t^c - f_0^c$ tracking the path of the exogenous fixed cost. An inactive planner is one for whom $\theta_{ss} = \theta_{cyc} = 0$.

The magnitude, ε, and persistence, α, of the fixed cost crisis are calibrated to the GFC. We assume exponential decay of the fixed cost shock:

$$f_t^c = f_0^c + \varepsilon \cdot \alpha^t, \quad \alpha \in (0,1). \tag{41}$$

We choose α such that the shock has a half-life of one year, and ε is calibrated such that 20.44% of firms exit on impact if the cycle policy is passive (steady state policy may be active). This figure corresponds to the percentage deviation from trend in the number of active firms in Spain during the crisis period.

[17]We calibrate β to the firm discount rate estimated in Gormsen and Huber (2025), which lies toward the lower end of empirical estimates. Our quantitative results are very similar under higher values of β.

	Value	Description	Target/Source
β	0.864	Firm discount rate (p.a.)	Gormsen and Huber (2025)
δ	0.0336	Death rate (p.a.)	average pre-GFC exit rate in SABI
β^*	0.894	Social planner discount factor (p.a.)	$\beta/(1-\delta)$
h	1.2	Tail parameter	revenue distribution of firms in SABI
σ	5.4	Elasticity of substitution	section 5
q	0.568	Love-of-variety	section 5
L^p	1.0	Labor supply	normalization
f_0^c	1.0	Steady state fixed cost	normalization
α	0.841	Shock decay	1 year half-life
ϵ	see text	Shock magnitude	deviation from trend in # of firms in SABI

Table 3: Parametrization of the quantitative model.

6.2 Results

We discuss the dynamics of output, Y, and its components: the mass of variety, M, aggregate productivity, $\mathcal{Z}$, and production labor, L^p, over the business cycle. We begin by considering the laissez-faire allocation, then a planner with only a steady-state instrument, and, finally, one that has both steady-state and cycle-specific instruments.

6.2.1 Laissez-Faire

We start by studying the laissez-faire scenario of Figure 6. From Panel B, it can be seen that on impact, the targeted 20.44% of varieties are lost, leading to a concurrent drop in aggregate productivity and production labor of 0.9% and 1.87%, respectively. As shown in Panel C, these adjustments jointly lead to a drop in output of 9.28% on impact, compared with a 2.75% contraction in the CES benchmark. While output in the CES economy experiences a small and stable long-term gain in output, the economy featuring our estimated q experiences a persistent fall in output relative to its pre-crisis steady-state level. In consumption equivalent variation (CEV), the recession cost is 5.28% of steady-state consumption.

6.2.2 Optimal Steady-State Subsidy

We consider an economy where a social planner implements the optimal steady-state intervention characterized in Proposition 6.III). Since $q > 1/(\sigma-1)$, the planner optimally subsidizes the fixed cost in steady state, accordingly. Quantitatively, we find that $\theta_{ss} = 0.88$.[18] Importantly, since q is substantially larger than $1/(\sigma-1)$, the economy is very far from first-best. Hence, the steady-state subsidy is very powerful and increases welfare by 55.30% in consumption-equivalent terms, relative to the laissez-faire steady state.

The recession dynamics in this economy can be observed in Figure 7. The initial output loss is substantially larger than in the laissez-faire economy, 12.04%, as shown in Panel C.

[18]Note that in Proposition 6, we solved the problem of a myopic planner, whereas here we solve a forward-looking Ramsey problem with discount rate β and exit rate δ. While the qualitative implication are the same, the numerical solution is different because of the different planning problem.

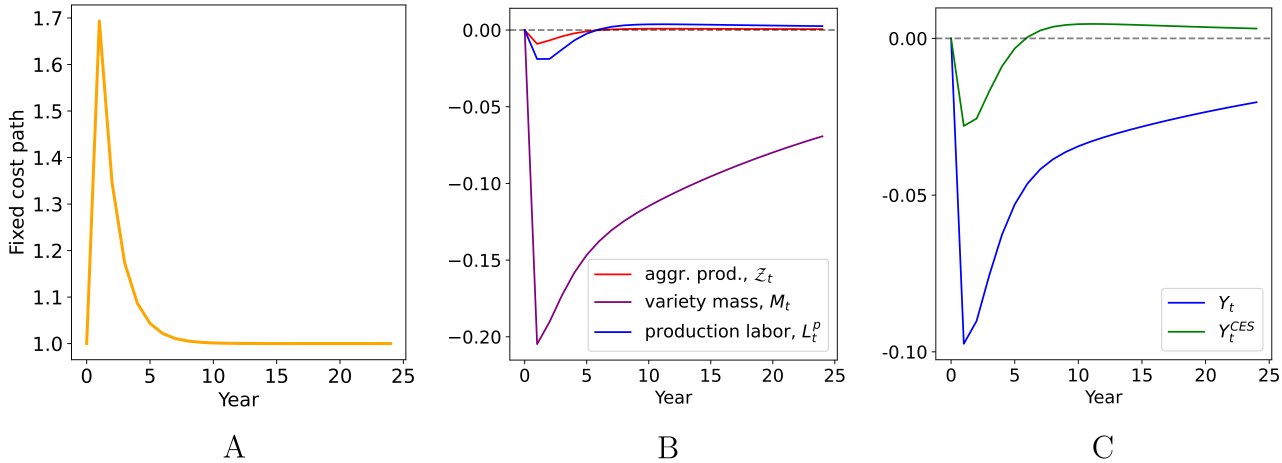

Figure 6: Fixed-cost shock path and impulse responses in a laissez-faire economy. Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.

This is driven by the higher equilibrium number of firms, which require more labor diversion as the fixed costs increase. After the initial decline, L^p recovers as fixed costs return to their original level. The steady-state subsidy implemented by the planner fundamentally changes the economy by making entry and continued existence cheaper for firms, while increasing the price of production labor. It exacerbates the crisis in the very short run, but causes the recovery to be faster. The long-run consequence is that the recession is a small increase in output in the long run, still associated with negative welfare effects due to the volatility and discounting. In CEV terms, the welfare costs of the cycle are 3.68% of steady-state consumption. This loss is mostly driven by the loss of varieties, which alone reduces welfare by 4.03%, which is only partially offset by small gains in production labor and TFP.

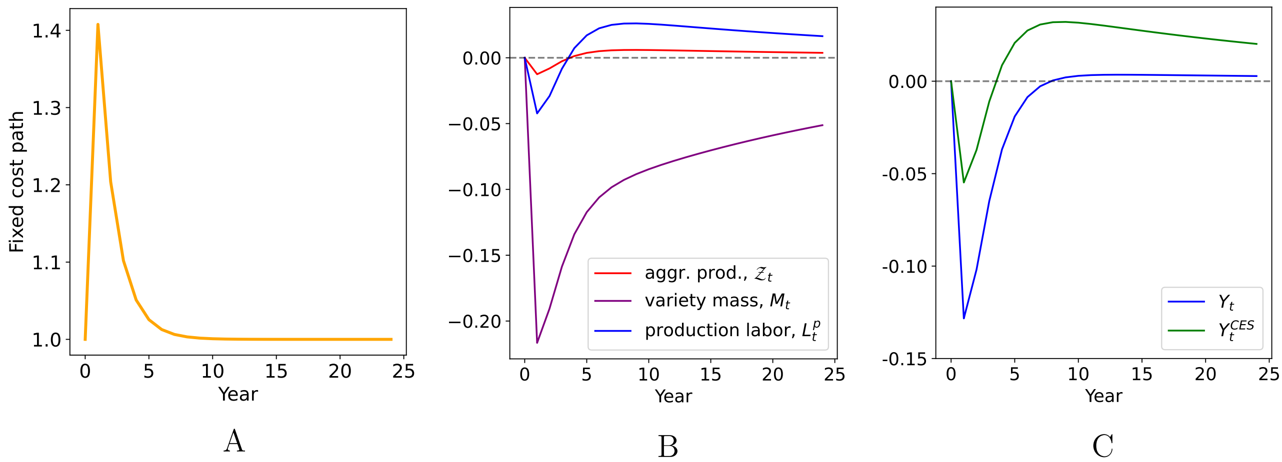

Figure 7: Fixed-cost shock path and impulse responses with a steady-state planner. Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.

6.2.3 Optimal Steady-State and Cycle Subsidy

Finally, we examine the dynamics if the planner has an optimal θ_{ss} policy in place before the shock, and supplements this with an optimal choice of θ_{cyc}.[19] In this setting, depicted in Figure 8, the social planner aims to undo variety losses entirely by heavily subsidizing the fixed cost, as shown in Panel B. The planner chooses $\theta_{cyc} > 0$ such that fixed cost after subsidies are lower than in steady state: $\bar{f}_t^c < (1 - \theta_{ss}) f_t^c$ as per Panel A. This causes the crisis to unfold almost exclusively in the factor markets. Since there is no reaction in firm exit, the fixed cost payments $M_t f_t^c$ shouldered by the economy become very large. As a consequence, labor allocated to production collapses. Accordingly, output is close to the CES benchmark in this scenario, see Panel C. The crisis is deep but short-lived. This is driven by the planner intervention: as the rise of fixed costs is accommodated by fiscal policy, the recovery is very fast since there is no active selection channel. Furthermore, since selection is the sole source of history-dependence, the economy returns to the pre-recession output level. Quantitatively, the recession with an active planner has a CEV welfare cost of 3.48%. We conclude that, relative to a planner constrained to steady-state policy, a planner that can respond contingently reduces the welfare cost of business cycles by 5%, from 3.68% to 3.48% CEV. In contrast to recessions with an inactive planner, most of the welfare loss originates from the decline in labor used in production (-1.84%), while the contribution of variety loss is comparatively limited, at -1.43% CEV.

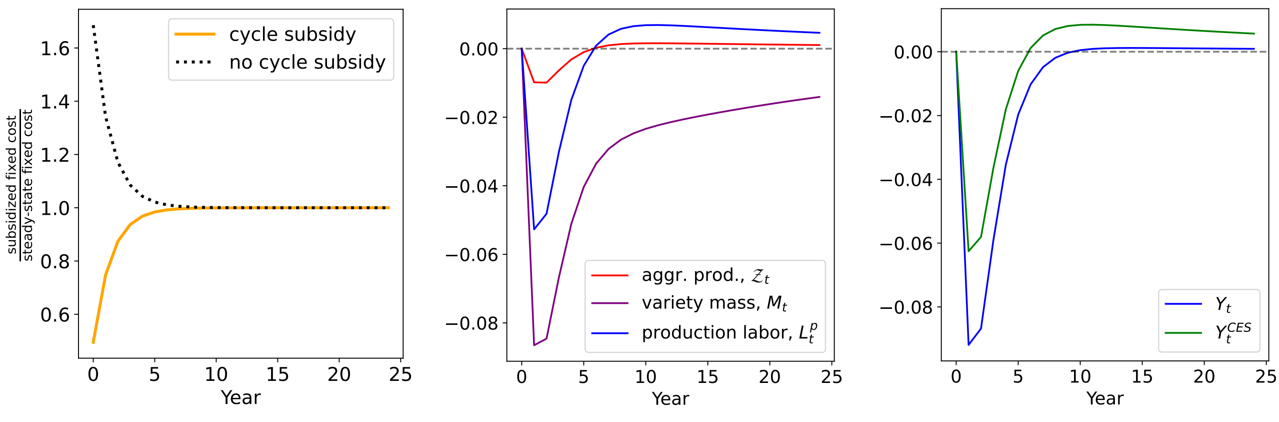

Figure 8: Fixed-cost shock path and impulse responses with a cycle planner (ss + cycle). Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.

We summarize the effects of policy and the welfare costs of recessions in our scenarios in Table 4. The equivalent of Figures 6, 7, and 8, and Table 4 with $q = 0.3$, as estimated by Baqaee et al. (2023), can be found in Appendix D. The results are qualitatively similar, with the caveat that, as love-of-variety is smaller, the economy is less inefficient and recessions are less costly.

[19] We analyze these policies jointly because if the planner did not have a steady-state subsidy in place, they would try to use the cycle response to induce long-run changes.

Table 4: Steady-state welfare gains and recession welfare costs (CEV, %)

A. Steady State	*CEV (%)*
ss policy	**+55.30%**

B. Recession	*CEV (%)*
Welfare cost of recession	**-5.28%**
Variety	-4.67%
Production labor	-0.40%
TFP	-0.21%
Welfare cost of recession with ss policy	**-3.68%**
Variety	-4.03%
Production labor	+0.31%
TFP	+0.05%
Welfare cost of recession with ss and cycle policy	**-3.48%**
Variety	-1.43%
Production labor	-1.84%
TFP	-0.22%

Notes: the top subsection reports the *steady-state* welfare gain from adopting the steady-state planner relative to a laissez-faire economy. All other rows report the *welfare cost of the recession* relative to the corresponding steady state. The decomposition of the total welfare cost shows the contribution of the unique components of output, i.e. Variety Mass, Production labor, and TFP, to the total CEV. The results are reported using the q and σ estimate in Section 5. Numbers are shown as CEV percentages.

7 Conclusions

We study the cleansing effect of recessions in an economy where agents value the presence of multiple differentiated varieties. We show that recessions driven by rising fixed costs of production generate *cleansing* in the sense that exiting firms are replaced by more productive entrants. However, we show that this need not translate into long-run gains in GDP and welfare. Whether the household is better off in the long run fundamentally depends on the extent of *love-of-variety* in the downstream aggregation technology for industry output. We show that when *love-of-variety* is stronger than in the benchmark CES economy, a social planner finds it optimal to increase fixed-cost subsidies during recessions to reduce the extent of firm exit. Our model-consistent estimates suggest that recessions generate long-run welfare losses over and above their transitional costs, contrary to what posited by the liquidationist view of business cycles.

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Online Appendix

Not-so-Cleansing Recessions

A Extensions and Additional Results

A.1 Homothetic Aggregators

In this section, we show that within the class of homothetic aggregators with variety externality, the aggregator in eq. (1) is particularly suited for analytical characterizations and empirical work.

Proposition A.1 (Homothetic Aggregators)**.** *Let $Y(\boldsymbol{y})$ be a homothetic production function from the HSA, HIIA, or HDIA families by Matsuyama (2025), acting on a bundle of inputs $\boldsymbol{y}$. Let $M = |\boldsymbol{y}|$ be the number of varieties in the input bundle. Suppose that*

1. *One can separate Y into a variety externality and an aggregator with constant, zero LoV:*

$$Y(\boldsymbol{y}) = h(M)g(\boldsymbol{y}) \quad \textit{with} \quad \frac{d\ln g}{d\ln M} = 0$$

2. *The variety externality has constant LoV:*

$$\frac{d\ln h}{d\ln M} = q = const.$$

Then Y is the aggregator in eq. (1), with $g(\boldsymbol{y}) = Y^{CES}(\boldsymbol{y})\, M^{-\frac{1}{\sigma-1}}$ and $h(M) = M^q$.

These desirable properties allow us to separate the role of love-of-variety from that of the price elasticity of demand sharply. They also allow us to recover empirically an exact map of our model, rather than a local effect in Section 5.

A.2 Entry Game Equivalence

In the main text, we have assumed that the entry process is an instantaneous equilibrium in which all market participants anticipate the firm distribution after entry, m. In an alternative equilibrium characterization without anticipation of m, firms iteratively enter the market, only considering the ex-ante distribution of firms in each iteration. Given a distribution of incumbents, initially, a small mass of firms enters the market, and the low-productivity subset folds immediately. This play repeats until no additional firm wishes to enter and no active firm wishes to quit. Proposition A.2 states that the two entry processes are equivalent.

Proposition A.2 (Entry game equivalence)**.** *The instantaneous equilibrium has a unique solution $(E, \underline{z}, m)$, which coincides with the limit point of the iterative entry game.*

A.3 Elastic Factor Supply

In the main text, we have considered economies with fixed factor supply. Here, we extend a positive result to economies with elastic factor supply.

Proposition A.3 (Elastic Factor Supply)**.** *Consider an economy with elastic primary factor supply (EF). Then, relative to an economy with fixed factor supply (FF), long-run output effects of a recession are dampened:* $|\Delta \log Y^{EF}| < |\Delta \log Y^{FF}|$.

A.4 Heterogeneous Externality Draws

In the main text, we study a setting in which firms draw idiosyncratic productivities upon entry but are symmetric in terms of their contribution to the variety externality: each firm contributes a single variety. Consider an extension of our baseline economy where firms draw their contribution ξ to the aggregate externality term. Suppose that, upon entry, entrants draw once and for all from a joint distribution $(\xi, z) \sim \mu^E(\xi, z)$. Denote the marginal entry distributions μ^E_ξ and μ^E_z, and the marginal distributions of active firms by m_ξ and m_z, for ξ and z, respectively. W.l.o.g., we assume that the expected externality contribution drawn by an entrant is unity: $\mathbb{E}_{\mu^E_\xi}[\xi] = 1$. The aggregate externality is then $\mathcal{M} \equiv \int\int \xi m(z, \xi)\ \mathrm{d}z\ \mathrm{d}\xi = \mathbb{E}_{m_\xi}[\xi] \times M$, with M being the count of how many firms there are in the market: $M = \int\int m(\xi, z)\ \mathrm{d}z\ \mathrm{d}\xi$. Importantly, as varieties have heterogeneous effects on the aggregator, we adopt $\mathcal{M}$ as the measure of varieties. Namely, LoV is the elasticity to $\mathcal{M}$. The only HSA aggregator which satisfies the separability conditions of Proposition A.1 is given by

$$Y = \mathcal{M}^q \times \left[\int y(z)^{(\sigma-1)/\sigma} \mu_z(z)\ \mathrm{d}z\right]^{\frac{\sigma}{\sigma-1}}. \tag{42}$$

Note that the first factor is isoelastic in $\mathcal{M}$ with elasticity q, while the second factor has zero elasticity with respect to the number of varieties, since μ_z is a marginal probability density.

Intuitively, heterogeneous externalities interact with the cleansing effects of cycles if and only if they are correlated with the selection mechanism based on productivity. We confirm this intuition in the following remark.

Remark A.1 (Independence)**.** *If ξ and z are independent in entrant and incumbent distribution, then the model is equivalent to our baseline model in firm-level choices and aggregates.*

Away from this special case, we characterize the economy with a general joint distribution through the cycle, where we again refer to pre-cycle, recession, and post-cycle values with indexes 1, 2, and 3, respectively. The average externality contribution in the economy along the cycle is given by

$$\mathbb{E}_{m_1}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_1], \tag{43}$$

$$\mathbb{E}_{m_2}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_2], \tag{44}$$

$$\mathbb{E}_{m_3}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_2] \times \frac{M_2}{M_3} + \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_1] \times \frac{E\mu_z(z \geq \underline{z}_1)}{M_3}. \tag{45}$$

The last equation states that the long-run average contribution depends on the average among firms that survived throughout the recession (first term) and the average contribution among new entrants post-recession (second term). Like in the baseline model, we have $M_3 < M_1$. However, this is no longer the externality-relevant quantity in the current model. Instead, we note that

$$\mathcal{M}_3 > \mathcal{M}_1 \quad \iff \quad \Delta \log \mathbb{E}_m[\xi] > -\Delta \log M. \tag{46}$$

Namely, the variety effect is positive if the loss in the number of firms is more than compensated for by an increase in the average externality per firm. Since entry and exit choices at the firm level depend on productivity through their expected profits, the correlation between ξ and z determines whether recessions are less or more costly relative to the independent case of Remark A.1.

Clearly, the welfare effect of the externality depends now on whether a business cycle selects out firms with relatively high or low contributions to the aggregate externality. We formalize this additional effect in the following proposition, which relates this extension to our baseline model and Proposition 1.

Proposition A.4 (Business Cycles with Heterogeneous Externalities)**.** *Consider the extension with heterogeneous contributions to the externality. Say that ξ and z are positively (negatively) correlated if $\mathbb{E}_{\mu^E}[\xi \mid z \geq a]$ is strictly increasing (decreasing) in a. Let Y^{hom} be output in our baseline model with homogeneous externalities. Then:*

1. *The cycle induces negative selection in ξ and $\Delta Y < \Delta Y^{hom}$ if ξ and z are negatively correlated.*

2. *The cycle induces positive selection in ξ and $\Delta Y > \Delta Y^{hom}$ if ξ and z are positively correlated.*

3. *The cycle induces no selection in ξ and the heterogeneity is irrelevant to output $\Delta Y = \Delta Y^{hom}$, if ξ and z are uncorrelated.*

A.5 Forward-Looking Exit

A straightforward extension of the model is one in which firms anticipate the mean-reverting path of an adverse fixed cost shock, allowing them to see and discount $f^c_{t+1}, f^c_{t+2}, \ldots$. However, they do not foresee the path of future entry, forecasting future profits using $\widetilde{\pi}(z, m_t)$. The decision to exit then hinges on

$$NPV(\{f^c_{t+k}\}_k) \gtreqless \widetilde{\pi}(z, m_t)/(1-\beta). \tag{47}$$

Accordingly, the cutoff jumps up less during a recession, while the dynamics of the model are unaffected.

A.6 Aggregate Dynamics

In this section, we briefly revisit the dynamics of an economy in which the fixed cost increases and slowly reverts to its original value, in order to characterize the transition behavior. For exposition in this section, we assume that μ^E is Pareto, i.e., $\mu^E(z) = \beta z_{min}^{\beta} z^{-(\beta+1)}$ with $\beta > \sigma - 1$, and that the initial fixed cost are $f_{-1}^c = 1$. Fixed cost increase to $\phi > 1$ in $t = 0$ and revert back according to

$$f_t^c = \phi^{\max\{0, 1-t\cdot\frac{1}{T^*}\}}, \quad (t = 0, 1, 2, ...) \tag{48}$$

where T^* is the number of fixed cost changes until complete reversion. Hence, ϕ parametrizes intensity and T^* parametrizes smoothness of the crisis. Since the equilibrium is attained instantly with any parameter change, and since each equilibrium would be a steady state in the absence of further parameter changes, we are effectively comparing a sequence of steady state equilibria. If one assigns unit length to the crisis, then $\mathrm{d}t = \frac{1}{T^*}$ is the length of a single time-delta. A subscript-t indicates the value of a variable after t such small intervals. Since the number of time-deltas passed through in a fixed cost cycle is T^*, $T^* \to \infty$ characterizes a continuous time limit. Using the equilibrium definition repeatedly, we derive closed-form solutions of the equilibrium at the end of the crisis (after T^* time-deltas). Solutions are given in the propositions below. We assume that the pre-crisis equilibrium with entry and cutoff—which we call E_{-1} and $\underline{z}_{-1}$, respectively—is derived from entry into a previously empty economy (i.e., no prior incumbents).

Proposition A.5 (Pareto Economy with Sequential Fixed Cost Reversion)**.** *After a fixed cost increase, during the recovery, $t \in \{1, ..., T^*\}$, the economy features a constant flow of entrants*

$$E_t = \frac{\mathcal{I}}{\beta}\frac{\sigma-1}{\sigma}\frac{1}{f^e}\left[1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right] \quad (t = 1, 2, ..., T^*). \tag{49}$$

The sequence of cutoff productivities, including the cutoff on impact in $t = 0$, is smoothly declining and given by

$$\underline{z}_t = z_{min}\left\{\frac{\phi^{(T^*-t)/T^*}}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{1/\beta} \quad (t = 0, 1, ..., T^*). \tag{50}$$

The measure of active firms after the crisis is given by

$$\begin{aligned} m_{T^*}(z) = {} & \frac{\mathcal{I}}{f^e}\frac{\sigma-1}{\sigma}\beta z_{min}^{\beta} z^{-(\beta+1)}\mathbb{I}\left\{z \geq z_{min}\left\{\frac{1}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{1/\beta}\right\} \\ & + \frac{\mathcal{I}}{f^e}\frac{\sigma-1}{\sigma}\beta z_{min}^{\beta} z^{-(\beta+1)}\sum_{t=1}^{T^*}\mathbb{I}\left\{z \geq z_{min}\left\{\frac{\phi^{(T^*-t)/T^*}}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{1/\beta}\right\}\left[1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right]. \end{aligned} \tag{51}$$

The long-run aggregate productivity, $\mathcal{Z}_{T^*}$, equals its pre-crisis value:

$$\mathcal{Z}_{T^*} = \mathcal{Z}_{-1}, \tag{52}$$

and the long-run mass of active firms is

$$M_{T^*} = E_{-1} p_E(\underline{z}_0) + \sum_{\tau=1}^{T^*} p_E(\underline{z}_\tau) E_\tau = \frac{\mathcal{I}}{\sigma} \frac{\beta - (\sigma - 1)}{\beta} \left\{ \phi^{-1} + (1 - 1/\phi) \left(\frac{1 - \phi^{-\frac{\beta-(\sigma-1)}{\beta}\frac{1}{T^*}}}{1 - \phi^{-\frac{1}{T^*}}} \right) \right\} \tag{53}$$

$$< \frac{\mathcal{I}}{\sigma} \frac{\beta - (\sigma - 1)}{\beta} = M_{-1}, \tag{54}$$

and all quantities approach their pre-crisis values as $\phi \downarrow 1$.

The characterization of the transition provides one important economic insight: the slow reversion of the fixed cost induces selection *along the way*. Since firms are allowed to enter before the fixed cost has reverted completely to f^c_{-1}, some of the post-crisis, entrants in period 2 face a more stringent cutoff than in period 3 and so on. This effect strengthens the cleansing effects and induces more long-run selection. The continuous time limit yields more succinct expressions in the following Proposition.

Proposition A.6 (Continuous Time Reversion)**.** *In the limit as $T^* \to \infty$, the mass of active firms converges from above to*

$$M_{T^*} \downarrow M_\infty = \frac{\mathcal{I}}{\sigma} \frac{\beta - (\sigma - 1)}{\beta} \left((1 - \phi^{-1}) \frac{\beta - (\sigma - 1)}{\beta} + \phi^{-1} \right). \tag{55}$$

Moreover, the firm distribution converges to

$$m_{T^*}(z) \to m_\infty(z) = \underbrace{\mu^E(z) \overbrace{\frac{\sigma - 1}{\beta} \frac{\mathcal{I}}{\sigma} \frac{1}{f^e}}^{=E_{-1}}}_{=m_{-1}(z)} \times \begin{cases} (\beta - (\sigma - 1)) \left[\ln z - \ln \underline{z}_{-1} \right] & z \in [\underline{z}_{-1}, \underline{z}_0), \\ \left[1 + \frac{\beta-(\sigma-1)}{\beta} \ln \phi \right] & z \in [\underline{z}_0, \infty). \end{cases} \tag{56}$$

The right tail of $m_{T^*}(z)$, $z \geq \underline{z}_0$ converges to $m_\infty(z)$ from below. The limit density $m_\infty(z)$ is continuous, except at $z = \underline{z}_0$, where it jumps up.

The continuous version discussed in Proposition A.6 reveals that cleansing effects in terms of an increased average productivity are more pronounced if fixed costs revert smoothly. Nonetheless, our main results still apply: If $q > q^{CES}$, then the long-run welfare effects of crises are negative. We illustrate graphically the result in Proposition A.6 in Figure A.1. As highlighted, the limiting distribution is different when the reversion is smooth. The distribution has zero mass at its lower truncation point, $\underline{z}_{-1}$. Such a shape is more in line with productivity distributions inferred from data. Second, the right tail thickens relative to m_{-1} by a factor that is increasing in the log of crisis intensity, yet the tail index of the distribution is unaffected.

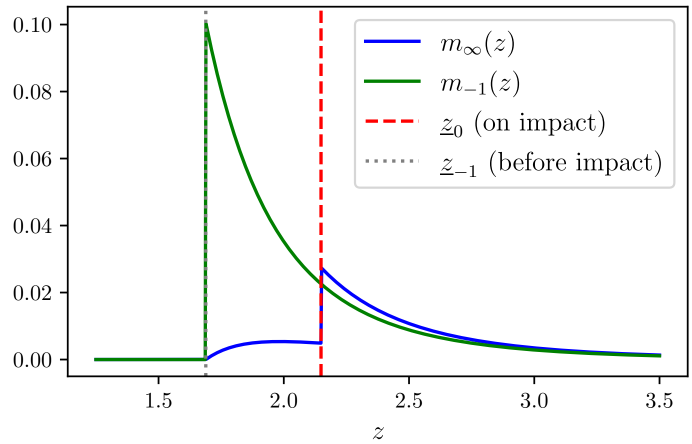

Figure A.1: Limit firm productivity distribution after smooth fixed cost reversion. The distribution $m_\infty(z)$ has zero mass at its lower truncation point and a thickened right tail, as described in Proposition A.6.

General Equilibrium Finally, we extend this characterization to the general equilibrium framework of Section 3. Because the smooth transition increases the strength of cleansing effects, the long-run number of firms is smaller. As a consequence, even less labor is used for fixed cost payments than in the case in which fixed costs revert all at once.

Proposition A.7 (Aggregate Dynamics in General Equilibrium). *In general equilibrium, entry subsides over multiple periods, and the steady state is attained in the limit as $E_t \to 0$. E_t exponentially decays, and the rate of decay is faster if successful entrants are more productive than firms at the cutoff, $\underline{z}$, and if the cutoff is low in the sense that $p_E(\underline{z})$ is close to 1.*

A.7 Stochastic Idiosyncratic Productivity

Our baseline model posits that firms draw their productivity once and for all. Here, we study a setting in which they experience fluctuations in idiosyncratic productivity even after entry. Suppose firms are subject to individual, transitory productivity shocks as in Hopenhayn (1992). Productivity shocks are i.i.d. across firms and periods. Firms decide at the beginning of the period, before learning their idiosyncratic shock, whether or not to produce. Effective productivity $\widetilde{z}$ consists of a fixed component, z, drawn from μ^E on entry, and a shock ε, such that $\widetilde{z} = z + \varepsilon$ and $\varepsilon \sim \psi$ with $\mathbb{E}[\varepsilon] = 0$.[20] We directly discuss the general equilibrium version of the model. Firm profits now read

$$\pi(\widetilde{z}, m) = \frac{R}{\sigma} \frac{(z+\varepsilon)^{\sigma-1}}{\widetilde{Z}(m,\psi)} - f^c, \tag{57}$$

where

$$\widetilde{Z}(m,\psi) := \int \int (z+\varepsilon)^{\sigma-1} m(z) \psi(\varepsilon) \, \mathrm{d}z \, \mathrm{d}\varepsilon \tag{58}$$

[20] For ease of notation, we assume that $z + \varepsilon \geq 0$ always holds. Similar results can be achieved if one parametrizes the distribution of ε by z to ensure $\varepsilon + z \geq 0$.

is market intensity, which nests our baseline model for $\psi = \delta_0$ (the Dirac distribution centered about 0). Given the timing assumption, the zero profit condition pins down the cutoff productivity. The equilibrium conditions describing firm entry and exit with idiosyncratic risk are given by

$$m_t(z) = m_{t-1}(z)\mathbb{I}_{\{z \geq \underline{z}_t\}} + E_{\varepsilon,t}\mu^E(z)\mathbb{I}_{\{z \geq \underline{z}_t\}}, \tag{59}$$

$$E_{\varepsilon,t}(\mathbb{E}_{\mu^E}[\max\{\mathbb{E}_\psi[\pi(z, m_t)], 0\}] - f^e) \leq 0, \tag{60}$$

$$E_{\varepsilon,t} \geq 0, \tag{61}$$

$$\mathbb{E}_\psi \pi(\underline{z} + \varepsilon, m_t) = 0, \tag{62}$$

and all other equilibrium equations are as in the baseline model. We note the following properties relative to our baseline model:

Proposition A.8 (Equilibrium with idiosyncratic risk)**.** *The following holds*

1. *As in the baseline model, the cutoff, $\underline{z}$ is history-independent in a state of entry.*
2. *An incumbent with permanent productivity component z is part of the steady-state if and only if the probability that temporary productivity fluctuations cannot make them exit at the peak of the crisis: $\mathbb{P}_\psi(z + \varepsilon < \underline{z}) = 0$. The pre-crisis and long-run allocations differ if and only if*

$$\int \mathbb{P}_\psi(z + \varepsilon < \underline{z})\mu^E(z) \, \mathrm{d}z > 0. \tag{63}$$

   A sufficient condition for both these to be true is that there exists a lower bound $\underline{\varepsilon}$ high enough.

An immediate consequence of Proposition A.8 is that whenever eq. (63) holds, Propositions 1–5, which all exploit the path dependency of our model, still hold true.

An alternative scenario is one in which *every* firm exits in some future, and incumbency plays no role in the steady state. For example, this would be the case whenever there are random exit shocks (Melitz, 2003; Bilbiie et al., 2012). With exogenous exit, the steady-state distribution becomes history-independent, and all the fixed cost cycle effects we discussed above become transitory. Nonetheless, the dynamics we described throughout would be visible through what Caballero and Hammour (1996) refer to as "echo" effects. Namely, persistent fluctuations caused by past recessions that only vanish in the very long run.

A.8 Multiproduct Firms

An important assumption of our model is that firms produce a single variety. As a consequence, firm exit necessarily implies the loss of a variety. Here, we study an extension of our baseline model where firms can produce multiple varieties. The key force that this extension introduces is that the exit of a firm may induce a surviving competitor to expand its variety set, leaving the number of offered goods in the economy unchanged. Effectively, this allows us to break the link between firm exit and variety losses.

Consider an economy in which individual firms pay a fixed cost $f^p > 0$ per variety they produce, and firm productivity is the same across varieties. Then, a firm with productivity z produces a total number of varieties $N(z)$. To aggregate varieties $j \in [0, N(z)]$ within a firm, we use a CES-aggregator with an elasticity of substitution $\eta > 1$:

$$y(z) = \left(\int_0^{N(z)} y_j(z)^{\frac{\eta-1}{\eta}} \, \mathrm{d}j \right)^{\frac{\eta}{\eta-1}}. \tag{64}$$

Varieties across firms are aggregated by a generalized CES aggregator with elasticity of substitution $\sigma \in (1, \eta)$. This aggregator features, in addition to the aggregate externality running through LoV for different firms, q_F, an externality term for LoV through products, q_V:

$$Y = M^{q_F - \frac{1}{\sigma-1}} \widetilde{N}^{q_V - \frac{1}{\eta-1}} \left[\int y(z)^{\frac{\sigma}{\sigma-1}} m(z) \, \mathrm{d}z \right]^{\frac{\sigma-1}{\sigma}}. \tag{65}$$

Here, the number of firms is $M := \int m \, \mathrm{d}z$, and $\widetilde{N}$ is the number of products per firm: $\widetilde{N} = N/M = \int N(z)\mu(z) \, \mathrm{d}z$. Since the cost to produce an extra variety is linear while the gains are concave, each firm chooses to operate an optimal mass $N(z) > 0$ of varieties:

$$N(z) = \frac{L^p}{(\eta-1)f^p} \frac{z^{\phi-1}}{\int z^{\phi-1} m(z) \, \mathrm{d}z}, \tag{66}$$

where $\phi - 1 := \frac{(\sigma-1)(\eta-1)}{\eta-\sigma}$. Like individual output in our baseline model, the number of varieties of each firm is increasing in its relative productivity $z^{\phi-1} / \left(\int z^{\phi-1} m(z) \, \mathrm{d}z \right)^{\frac{1}{\phi-1}}$ and decreasing in the fixed cost f^p. By aggregation, the total number of varieties is given by

$$N = \frac{L^p}{(\eta-1)f^p}, \tag{67}$$

and therefore varieties scale linearly in the amount of available production labor. Equivalently, the ratio of labor expended on product fixed cost to that used for production, Nf^p/L^p is constant and given by $1/(\eta-1)$. Intuitively, firms trade off allocating labor to producing more varieties versus more output of existing varieties. Since firms behave as if they lived in a world with CES aggregation, this trade-off is independent of scale, and Nf^p/L^p is a constant fraction. To solve for L^p, we use the labor market clearing condition:

$$L^p + Mf^c + Ef^e + Nf^p = \bar{L}, \tag{68}$$

from which we conclude

$$L^p = \frac{\eta-1}{\eta}(\bar{L} - Mf^c) \qquad \text{and} \qquad N = \frac{(\bar{L} - Mf^c)}{\eta f^p}. \tag{69}$$

As before, we consider the steady state, i.e., $E = 0$. By substituting the equilibrium production of each firm into the aggregator in eq. (65), we can write aggregate output in equilibrium as

$$Y \propto \overbrace{M^{q_F - q_V - \frac{1}{\phi-1}}}^{=M^{\left(q_F - \frac{1}{\sigma-1}\right)-\left(q_V - \frac{1}{\eta-1}\right)}} \quad (\bar{L} - Mf^c)^{1+q_V} \overbrace{\left(\int z^{\phi-1} m(z)\ \mathrm{d}z\right)^{\frac{1}{\phi-1}}}^{=:\mathcal{Z}}, \tag{70}$$

suppressing positive constants depending on η and f^p only. Contrary to eq. (8), both the mass of firms and the mass of varieties matter for aggregate output, and we allow for two separate parameters capturing how much each is valued: q_F and q_V, respectively. From eq. (70), it is clear that cleansing effects are qualitatively the same, with a few remarks. First, for $q_V > 0$, there are increasing returns to scale to *uncommitted labor*, $\bar{L} - Mf^c$, because more uncommitted labor increases both the number of varieties per firm $\widetilde{N}$, and firm output $y(z)$. Second, an increase in the number of firms M is valued through two channels: a mechanical contraction of varieties per firm $\widetilde{N}$ with elasticity $-(q_V - \frac{1}{1-\eta})$ and an expansion of firm-variety with elasticity $(q_F - \frac{1}{\sigma-1})$.

With appropriately redefined boundaries for the inequalities in Propositions 1–5, all our results on cleansing through fixed-cost cycles hold true. In particular, firm-level profits are

$$\pi(z, m) = \frac{L^p}{\phi - 1} \frac{z^{\phi-1}}{\int z^{\phi-1} m(z)\ \mathrm{d}z} - f^c, \tag{71}$$

eqs. (4), (5) and (3) yield exactly the same equilibrium conditions and entry/exit dynamics by replacing σ with ϕ to account for within-firm variety substitution. Note that f^p does not enter the profits of a firm. This is driven by the fact that changes in the cost of holding products make all firms shrink or expand their number of varieties, thereby not altering the effective relative productivities. As a consequence, movements in f^p do not induce exit or entry since all firms move in lockstep and profits are unchanged. We formalize the long-run effects of business cycles driven by firm-level fixed costs f^c in part (i) and driven by product-level fixed costs in part (ii) of Proposition A.9.

Proposition A.9 (Fixed-cost Cycles with Multi-Product Firms)**.** *In the presence of multi-product firms, the following holds true.*

(i) For recessions driven by temporary increases in f^c, where $q_F^{CES} := \frac{1}{\sigma-1}$ and $q_V^{CES} := \frac{1}{\eta-1}$:

(a) an analogue of Proposition 1 holds with

$$M_3 < M_1,\ L_3^p = L_1^p,\ N_3 = N_1,\ \mathcal{Z}_3 = \mathcal{Z}_1$$

and

$$\frac{Y_3}{Y_1} \gtreqless 1 \Leftrightarrow q_F - q_F^{CES} \lesseqgtr q_V - q_V^{CES};$$

(b) an analogue of Proposition 4 holds with

$$M_3 < M_1,\ L_3^p > L_1^p,\ N_3 > N_1,\ \mathcal{Z}_3 > \mathcal{Z}_1$$

and there exists a unique $q_F^\star$ *with*

$$q_F^\star - q_F^{CES} > q_V - q_V^{CES},$$

such that

$$\frac{Y_3}{Y_1} \gtreqless 1 \iff q_F \lesseqgtr q_F^\star;$$

(c) an analogue of Proposition 5 holds with

$$\exists!\ (q_\circ, q^\circ) \ \text{ with } \ q_\circ - q_F^{CES} > q_V - q_V^{CES}$$

such that long-run effects depend on f_h^c *intensity.*

(ii) Recessions driven by temporary increases in f^p *have no long-run effects.*

The temporary increase in firm-level fixed costs induces exit on impact. In partial equilibrium, surviving firms temporarily increase their number of products. As the fixed cost reverts to its initial level, incumbents' product set shrinks while entrants join the economy. Overall, because the recession has cleansing effects, the total number of firms is lower, but the number of products per firm is unchanged. The long-run effects on output and welfare depend on whether LoV is high enough to compensate the loss of varieties, yielding a condition similar to that of Proposition 1 but accounting for both within and across firms substitutability. In GE, as fewer firms operate after the recession, labor is relatively cheaper due to smaller firm fixed costs payments. This increases the optimal number of products for surviving firms. Like in Proposition 4, there is a unique level of LoV that makes agents indifferent between going through a fixed cost cycle or not. This level now accounts for the partial recovery of lost varieties by exiting firms, since incumbents increase their product range after the recession.

Finally, the setup of this extension also allows us to examine a second kind of fixed cost cycle: a cycle in f^p. Part (ii) states that f^p cycles are akin to aggregate TFP cycles in that they have no long-run effects. The intuition is as follows. First, note that, differently from firm entry, product creation does not require a sunk investment, hence there is no notion of 'product-incumbency'. Since we consider temporary increases in f^p, they only have long-run effects if they affect the firm distribution by inducing entry or exit. Firms reduce their product set while the recession occurs and create new products as f^p returns to the original level; however, since all firms reduce their product set at the same time, there is no effect on market shares and, therefore, on profits. As a consequence, this recession does not feature exit, but rather just a reduction in the number of products per firm. This is because, in our economy, the presence of multiple products operates like a shifter to the productivity of each individual firm: if $z' > z$, the effective productivity difference is even larger since the z' firm chooses to operate more products than the z firm. However, the elasticity of $N(z)$ to f^p is independent of z. Hence, when f^p increases, all firms reduce the product set by the same percentage. This operates like a reduction in aggregate productivity and does not affect the relative productivity of firms. As in Proposition 2, this shift does not induce any exit and, therefore, does not alter the firm distribution before and after the recession. As a consequence, the economy reverts to the initial steady state.

A.9 Fixed costs in output and labor units

To preserve efficiency in the decentralized economy with $q = q^{CES}$, we have assumed that fixed and entry costs are paid entirely in units of labor (Barro and Sala-i Martin, 2004). To generalize our model, we let these costs be produced with the following Cobb–Douglas technology combining output (share α) and labor (share $1-\alpha$):

$$B(Y, L) \;=\; \kappa(\alpha)\, Y^{\alpha} L^{1-\alpha}, \qquad \kappa(\alpha) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}, \quad \alpha \in [0,1].$$

This technology nests two special cases: $\alpha = 0$, in which costs are paid entirely in units of labor as in our model, and $\alpha = 1$, in which they are paid entirely in units of output. Throughout, we make the technical assumption that $\sigma > 1 + \alpha$. The following Proposition extends Proposition 4 to this general setting.

Proposition A.10 (Cleansing Effects of Cycles)**.** *The change in output after the crisis, when the fixed costs are produced by a Cobb-Douglas bundle of output (share α) and labor (share $1-\alpha$), is given by*

$$\Delta \log Y(q,\alpha) = \frac{\sigma-\alpha}{\sigma-1}\,\Delta \log\left(\int z^{\sigma-1} m(z)\;\mathrm{d}z\right) + (1-\alpha)\,(\,q - q^{\mathrm{CES}}\,)\,\Delta \log M. \tag{72}$$

For each $\alpha \in [0,1]$ there exists a value of love-of-variety $q^\star(\alpha)$ such that the crisis leaves output unchanged: $\Delta \log Y(q^\star(\alpha),\alpha) = 0$; with $q^\star(\alpha) \geq q^{\mathrm{CES}}$, with equality for $\alpha = 1$. Furthermore, output is decreasing in q around $q^\star(\alpha)$: $\frac{\partial}{\partial q}\Delta \log Y(q,\alpha)\Big|_{q=q^\star(\alpha)} < 0$. Which implies

$$q \gtrless q^\star(\alpha) \Rightarrow \Delta \log Y(q) \lesseqgtr 0 \quad \textit{for } q \textit{ in a neighborhood of } q^\star(\alpha)\textit{, to first order.}$$

The main results continue to hold when fixed costs are denominated in any combination of labor and output units. When the economy places greater weight on variety than under the CES benchmark ($q > q^\star(\alpha) \geq q^{CES}$), recessions reduce long-run output and welfare.

The intuition behind this result is as follows. The fixed-cost cycle shifts the economy from the original steady state to a new one in which the mass of active firms is lower. Aggregate output adjusts according to equation (72), which depends on both the number of varieties and the firm productivity distribution. After the recession, the mass of firms shrinks, while the surviving firms are on average more productive.

When $\alpha = 0$, fixed costs are paid entirely in labor units, and thus q does not affect the resources available to firms for production. The equilibrium trade-off then operates purely between variety and selection: for $q > q^\star$, the economy values variety more than average productivity, so the variety effect dominates the selection effect.

When $\alpha > 0$, part of the fixed cost is denominated in units of output. A higher q encourages entry, which increases output through the love-of-variety channel but also raises the nominal burden of fixed costs, reducing the scale of production among the most productive firms. When σ is large, goods are close substitutes and concentrating production in the most efficient firms is particularly valuable. If $q > q^\star(\alpha)$, however, the economy places excessive weight on preserving variety, diverting capacity to sustain additional firms rather than allowing efficient producers to expand. This shift of production away from the strongest margin under high σ implies that recessions reduce long-run output and welfare whenever $q > q^\star(\alpha)$.

A.10 Additional Results on Depth of Recessions

Panel (A) shows how CES output, Y^{CES}, and the total mass of firms, M_3, respond to different increases in the fixed costs, f_h^c. Both quantities are independent of q. Relative to before the crisis, the mass M_3 drops, reaching a minimum at depth $f_h^{c,*}$, where all the incumbents with productivity less than the average successful entrant have been replaced. CES output always increases relative to before the cycle and reaches its highest level when no further returns from labor cleansing and selection effects can be extracted, i.e., at $f_h^{c,*}$. As the fixed cost rises toward eliminating every firm during phase 2, all quantities return to their pre-crisis levels.

Next, panel (B) shows the evolution of the love-of-variety effect for different levels of q. When $q > q^{CES}$, this effect tracks the path of the number of firms in panel (A). When $q < q^{CES}$, the LOV externality is such that the economy is better off when fewer firms are active, so the effect mirrors that of M_3/M_1.

Finally, panel (C) shows the behaviour of output for different $q-$economies. When q is large, recessions are unambiguously welfare reducing and, vice versa, when q is small they are always cleansing. In the intermediate range, some recessions can induce welfare gains while others generate welfare losses depending on their depth.

This set of intermediate $q-$economies is characterized by an inference problem. In fact, past data on recessions cannot inform on whether future recessions will have cleansing effect or not.

B Proofs

Proof of Remark 1. Suppose wlog that z_i is increasing in the firm index, $i \in [0, M]$. We can then perform the change of measure:

$$V_M(\{y_i\}_{i\in[0,M]}) := M^{q-\frac{1}{\sigma-1}} \left(\int_0^M y_i^{\frac{\sigma-1}{\sigma}} \, \mathrm{d}i\right)^{\frac{\sigma}{\sigma-1}} = M^{q-\frac{1}{\sigma-1}} \left(\int_{z\in\mathcal{Z}} y(z)^{\frac{\sigma-1}{\sigma}} m(z) \, \mathrm{d}z\right)^{\frac{\sigma}{\sigma-1}},$$

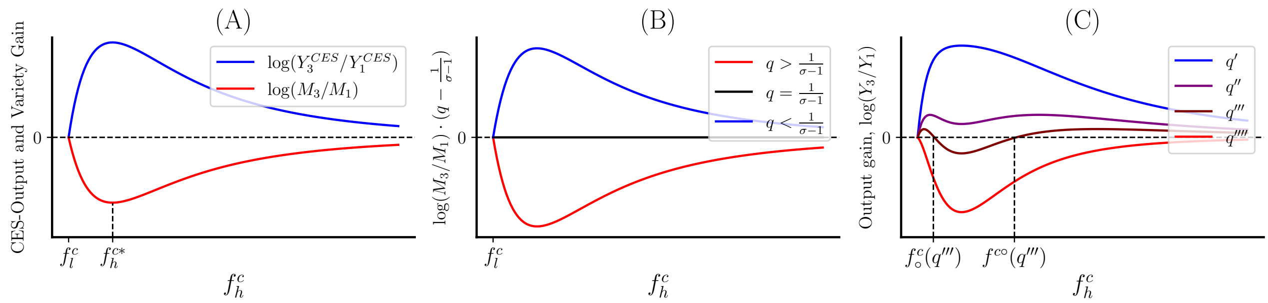

Figure A.2: Decomposition of the log-output ratio for varying levels of LoV, q and crisis, f_h^c. All curves start in $(f_l^c, 0)$ and are simulated from the model. Panel (A) shows the how $\ln(Y_3^{CES}/Y_1^{CES})$ and $\ln(M_3/M_1)$ vary with f_h^c. Panel (B) shows how LOV contributes to the output ratio through the factor $\ln((M_3/M_1)^{q-q^{CES}})$. Panel (C) sketches the curve of $\ln(Y_3/Y_1)$ for values of q, $q' < q'' < q''' < q''''$, whereby only $q''' \in (q_\circ, q^\circ)$ holds (cf. Proposition 5). Correspondingly, at q''', the effect of crises is ambiguous, and there are two crises $f_\circ^c(q''') < f^{c\circ}(q''')$ at which the output ratio returns equals 1. A proof of the 'wiggly' shape when LoV is q''' is provided in the proof of Proposition 5.

where the right-hand side is our aggregator. As in Benassy (1996), we define

$$v(M) := \frac{V_M(1)}{M} = \frac{M^{q-\frac{1}{\sigma-1}} M^{1+\frac{1}{\sigma-1}}}{M} = M^q.$$

The function $v(M)$ characterizes the relative "gain derived from spreading a certain amount of production between M differentiated products instead of concentrating it on a single variety" (Benassy, 1996). The elasticity of $v(M)$ with respect to M is defined as LoV, which indeed equals q. ■

Proof of Lemma 1. We note that the existence of the incumbent density function m_{t-1} does not matter for the determination of the cutoff. To see this, define profits gross of the fixed cost $\widetilde{\pi}(z, m_t) = \pi(z, m_t) + f^c$ and recall that the marginal firm is pinned down by the zero-profit condition

$$\widetilde{\pi}(\underline{z}_t, m_t) = f^c \tag{73}$$

Furthermore, if the free entry condition holds exactly, we have

$$f^e = \int_{a \geq \underline{z}_t} (\widetilde{\pi}(a, m_t) - f_c)\mu^E(a)da \tag{74}$$

Using the ZPC we obtain

$$f^e = \int_{a \geq \underline{z}_t} (\widetilde{\pi}(a, m_t) - \widetilde{\pi}(\underline{z}_t, m_t))\mu^E(a)da \tag{75}$$

which can be rewritten as

$$\frac{f^e}{f^c} = \int_{a \geq \underline{z}_t} \left(\frac{\widetilde{\pi}(a, m_t)}{\widetilde{\pi}(\underline{z}_t, m_t)} - 1 \right) \mu^E(a)da \tag{76}$$

Finally, we note that under CES and monopolistic competition, the relative gross profits $\frac{\tilde{\pi}(a,m_t)}{\tilde{\pi}(\underline{z}_t,m_t)}$ are equal to the relative market sizes $\frac{r(a,m_t)}{r(\underline{z}_t,m_t)}$ because the profit rates are constant and equal to $1/\sigma$. Furthermore, since markups are identical across firms, the relative size of firms is given by $\frac{r(a,m_t)}{r(\underline{z}_t,m_t)} = \left(\frac{a}{\underline{z}_t}\right)^{\sigma-1}$, which implies

$$\frac{f^e}{f^c} = \int_{a \geq \underline{z}_t} \left[\left(\frac{a}{\underline{z}_t}\right)^{\sigma-1} - 1\right] \mu^E(a) da. \tag{77}$$

Since the identity of the marginal firm only depends on the market intensity and the market intensity, in a state of entry, is invariant to the incumbent distribution, we have that $\underline{z}_t$ does not depend on m_{t-1}.

■

Proof of Proposition 1. Start by noticing that

$$\int z^{\sigma-1} m_3(z) \ \mathrm{d}z = \int z^{\sigma-1}\left(E_1 \mu^E \mathbf{1}_{\{z \geq \underline{z}_2\}} + E_3 \mu^E \mathbf{1}_{\{z \geq \underline{z}_3\}}\right) \mathrm{d}z \tag{78}$$

$$= E_1 \int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z + E_3 \int_{\{z \geq \underline{z}_3\}} z^{\sigma-1} \mu^E \ \mathrm{d}z \tag{79}$$

$$= E_1 \int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_{\{z \geq \underline{z}_3\}} z^{\sigma-1} \mu^E \ \mathrm{d}z \tag{80}$$

$$= E_1 \int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z \tag{81}$$

$$= \int z^{\sigma-1} m_1(z) \ \mathrm{d}z. \tag{82}$$

Therefore, the factor $\int z^{\sigma-1} m_\tau \ \ \mathrm{d}z$ remains unchanged in before and after the cycle. Next, consider some $a \geq \underline{z}_2 > \underline{z}_1$, then

$$\frac{\int_a^\infty m_3(z) \ \mathrm{d}z}{\int m_3(z) \ \mathrm{d}z} = \frac{E_1 \int_a^\infty \mu^E(z) \ \mathrm{d}z + E_3 \int_a^\infty \mu^E(z) \ \mathrm{d}z}{\int m_3(z) \ \mathrm{d}z} \tag{83}$$

$$= \frac{E_1 \int_a^\infty \mu^E(z) \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_a^\infty \mu^E(z) \ \mathrm{d}z}{\int m_3(z) \ \mathrm{d}z} \tag{84}$$

$$= \frac{E_1 \int_a^\infty \mu^E(z) \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_a^\infty \mu^E(z) \ \mathrm{d}z}{E_1 \int_{\underline{z}_2}^\infty \mu^E(z) \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_{\underline{z}_1}^\infty \mu^E(z) \ \mathrm{d}z} \tag{85}$$

$$> \frac{E_1 \int_a^\infty \mu^E(z) \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_a^\infty \mu^E(z) \ \mathrm{d}z}{E_1 \int_{\underline{z}_1}^\infty \mu^E(z) \ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E \ \mathrm{d}z}\right) \int_{\underline{z}_1}^\infty \mu^E(z) \ \mathrm{d}z} \tag{86}$$

$$= \frac{\int_a^\infty \mu^E(z)\ \mathrm{d}z}{\int_{\underline{z}_1}^\infty \mu^E(z)\ \mathrm{d}z} \tag{87}$$

$$= \frac{\int_a^\infty m_1(z)\ \mathrm{d}z}{\int m_1(z)\ \mathrm{d}z}. \tag{88}$$

Likewise, for $\underline{z}_1 \leq a < \underline{z}_2$

$$1 - \frac{\int_a^\infty m_3(z)\ \mathrm{d}z}{\int m_3(z)\ \mathrm{d}z} = \frac{\int_0^a m_3(z)\ \mathrm{d}z}{\int m_3(z)\ \mathrm{d}z} \tag{89}$$

$$= \frac{E_1 \int_a^a \mu^E(z)\ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}\right) \int_{\underline{z}_1}^a \mu^E(z)\ \mathrm{d}z}{E_1 \int_{\underline{z}_2}^\infty \mu^E(z)\ \mathrm{d}z + E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}\right) \int_{\underline{z}_1}^\infty \mu^E(z)\ \mathrm{d}z} \tag{90}$$

$$\leq \frac{E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}\right) \int_{\underline{z}_1}^a \mu^E(z)\ \mathrm{d}z}{E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E\ \mathrm{d}z}\right) \int_{\underline{z}_1}^\infty \mu^E(z)\ \mathrm{d}z} \tag{91}$$

$$= \frac{\int_0^a m_1(z)\ \mathrm{d}z}{\int m_1(z)\ \mathrm{d}z} = 1 - \frac{\int_a^\infty m_1(z)\ \mathrm{d}z}{\int m_1(z)\ \mathrm{d}z}, \tag{92}$$

so that again $\frac{\int_a^\infty m_3(z)\ \mathrm{d}z}{\int m_3(z)\ \mathrm{d}z} \geq \frac{\int_a^\infty m_1(z)\ \mathrm{d}z}{\int m_1(z)\ \mathrm{d}z}$. Thus, the probability density function $\frac{m_3(z)}{\int m_3(z)\ \mathrm{d}z}$ strictly first-order dominates the probability density function $\frac{m_1(z)}{\int m_1(z)\ \mathrm{d}z}$. Since $g(z) = z^{\sigma-1}$ is increasing for $\sigma > 1$, FOSD implies

$$\frac{\int z^{\sigma-1} m_3(z)\, dz}{\int m_3(z)\, dz} > \frac{\int z^{\sigma-1} m_1(z)\, dz}{\int m_1(z)\, dz}. \tag{93}$$

Combining this with the first part of this proof, we obtain $\int m_1(z)\ \mathrm{d}z > \int m_3(z)\ \mathrm{d}z$.

■

Proof of Proposition 2. We write the equilibrium conditions for an economy with aggregate productivity shock A as below and index all endogenous quantities by A:

$$f^e \geq \int_{\underline{z}_A} \left[\frac{R_A}{\sigma} \frac{(Az)^{\sigma-1}}{\int_{\underline{z}_A} (Az)^{\sigma-1} m_A(z)\ \mathrm{d}z} - f^c \right] \mu^E(z)\ \mathrm{d}z \tag{94}$$

$$0 = \frac{R_A}{\sigma} \frac{(A\underline{z}_A)^{\sigma-1}}{\int_{\underline{z}_A} (Az)^{\sigma-1} m_A(z)\ \mathrm{d}z} \tag{95}$$

Note that A cancels in both equations above, such that the system is equivalent to the equilibrium equations of our baseline model if and only if $R_A = R$. To establish this equality, note

$$R_A = \bar{L} + \Pi_A$$

$$= \bar{L} + R_A/\sigma - \left(\int m_A(z)\ \mathrm{d}z \right) f^c \quad \text{(integrate } \pi_A(z, m)\text{)}. \tag{96}$$

Eqs. (94–96) are then exactly the same as in the baseline equilibrium, and thus any $(m, \underline{z}, R)$ satisfying the baseline equilibrium also satisfy (94–96). We can give more detail. Let m^I be the predetermined incumbent density (in a dynamic context, $m_t^I = m_{t-1}$). In the case with entry, use $m_A = (E_A\mu^E + m^I)\mathbb{I}_{z\geq \underline{z}_A}$ in eqs. (94–95) to derive eq. (7), so $\underline{z}_A = \underline{z}$. Use eq. (96) in (95) and the definition of m_A to see that $E_A = E$, hence $m_A = m$. Clearly, then also $R_A = R$. In the case without entry, $E_A = 0 = E$ and eq. (95) alone pins down $\underline{z}_A$, after substituting for m_A. Since this equation is again independent of A, $\underline{z}_A = \underline{z}$.

■

Proof of Proposition 3. First, note that the presence of a random exit shock δ implies that the stationary distribution of firms is unique and so is the number of firms M^{ss}. As a consequence, there is a unique steady-state cutoff $\underline{z}^{\text{ss}}$.

Without loss of generality, consider a case in which the entry cost increases and then steadily declines.

Suppose that, for any arbitrary temporary increase in f^e, the cutoff $\underline{z}$ is non-increasing. Let m^I be the predetermined incumbent density (in a dynamic context, $m_t^I = m_{t-1}$), and μ^I be the corresponding pdf. Then there are two cases

1. f^e is large enough that the free entry condition is slack, and there is an initial part of the transition where $E = 0$, the number of firms steadily declines until some new level M' such that the free entry condition holds exactly. From this point onwards, any further reduction in f^e is associated with a strictly increasing number of firms until the entry cost returns to its initial level and $M \to M^{\text{ss}}$ from below

2. f^e increases but not enough to offset the exit rate delta, $E > 0$, along the entire transition and $M \to M^{\text{ss}}$ from below.

Consider now the behaviour of the cutoff. Again, we consider the same two cases

1. If the increase is large enough that $E = 0$ in the early periods of the transition, we know from the exit case above that the cutoff remains constant, since it is determined by

$$f^c = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^I(z)\,\mathrm{d}z}$$

   which is independent of f^e.

   As soon as the entry cost declines enough to have $E > 0$, the cutoff is determined by

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty} \left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1} - 1\right] \mu^E(z)\,\mathrm{d}z$$

   which implies that the cutoff jumps to some new level $\underline{z}' < \underline{z}$ since the RHS is strictly decreasing in $\underline{z}$. As the fixed entry cost keeps decreasing back towards the original level, $\underline{z}'$ converges to $\underline{z}^{\text{ss}}$ from below.

2. If the entry cost increase is small enough that $E > 0$ throughout the transition, then the cutoff jumps to some level $\underline{z}' < \underline{z}$ on impact and then converges back to $\underline{z}^{ss}$ from below.

This verifies the original guess and proves that along the transition, the cutoff decreases and so does the number of firms.

To prove part *c)*, note that since both $\underline{z}$ and M decrease along the transition and since exit is *unselected*, the average productivity of firms has to decrease as well. Since M and $\underline{z}$ both decline, then output has to decrease independently of q. ■

Proof Proposition 4. Note first that in the absence of fixed cost of entry, f^e, eq. (15) determines E^{ss} directly: there is no additional labor freed up from lower entry expenses. Hence, the entry process terminates immediately and jumps to E^{ss}. Rewrite eq. (15) now as

$$E_3 = \frac{R_3}{\sigma f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} - E_1 \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \tag{97}$$

$$= \frac{R_1}{\sigma f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} - E_1 \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} + (R_3 - R_1) \frac{1}{\sigma f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \tag{98}$$

$$= E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \right) + (R_3 - R_1) \frac{1}{\sigma f^c} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \tag{99}$$

where R_τ is steady-state GE income (i.e., without fixed cost of entry). Note that

$$(R_3 - R_1) = \frac{\sigma}{\sigma - 1} f^c (E_1 p_E(\underline{z}_1) - (E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1))) \tag{100}$$

$$= \frac{\sigma}{\sigma - 1} f^c (M_1 - M_3). \tag{101}$$

Suppose that $(R_3 - R_1) \leq 0$, then $M_3 \geq M_1$. Since $E_3 \leq E_1 \left(1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \right)$, following the steps of the proof of Proposition 1, we know that $\int z^{\sigma-1} m_3(z) \, \mathrm{d}z \leq \int z^{\sigma-1} m_1(z) \, \mathrm{d}z$. But then, it also holds that $\int z^{\sigma-1} \mu_3 \, \mathrm{d}z \leq \int z^{\sigma-1} \mu_1 \, \mathrm{d}z$, i.e., the average productivity of a firm active in the market is lower post cycle. By construction of the exit and entry process (clipping the left tail of the productivity distribution in period 2 and allowing entry of average firms in period 3) this cannot be. Hence, $(R_3 - R_1) > 0$ and thus

$$M_3 < M_1, \text{ and } \int z^{\sigma-1} m_3(z) \, \mathrm{d}z > \int z^{\sigma-1} m_1(z) \, \mathrm{d}z. \tag{102}$$

Consequently, $L_3^p > L_1^p$ holds, too. The output ratio between pre- and post-crisis steady-state depending on q is then given by

$$\frac{Y_3}{Y_1}(q) = \left[\frac{M_3}{M_1} \right]^{q - \frac{1}{\sigma-1}} \left[\frac{L_3^p}{L_1^p} \right] \frac{(\int z^{\sigma-1} m_3(z) \, \mathrm{d}z)^{1/(\sigma-1)}}{(\int z^{\sigma-1} m_1(z) \, \mathrm{d}z)^{1/(\sigma-1)}}, \tag{103}$$

and none of the equilibrium quantities on the right-hand-side depend on q. It follows that

$\frac{Y_3}{Y_1}\left(\frac{1}{\sigma-1}\right) > 1$ for the CES case, and there exists a unique $q^* > \frac{1}{\sigma-1}$ for which $\frac{Y_3}{Y_1}(q^*) = 1$. ■

Proof of Proposition 5 and Figure A.2. We prove the characteristics of Figure A.2, panels A and B, first. To this end, note that $\underline{z}_2^{\sigma-1}$ is a strictly increasing, monotonic function of f_h^c defined by the zero profit condition (5), with $\underline{z}_2^{\sigma-1} \to \infty$ as $f_h^c \to \infty$, and any analysis in terms of $\underline{z}_2^{\sigma-1}$ carries over in terms of f_h^c. Now note that

$$\begin{aligned}
M_3 &= E_1 p_E(\underline{z}_2) + E_3 p_E(\underline{z}_1) \text{ and} \\
E_3 &= \left(1 + \frac{\underline{z}_1^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}_1](\sigma-1)}\right)^{-1} \\
&\quad E_1 \left[1 - \frac{\int_{\{z \geq \underline{z}_2\}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z} + (p_E(\underline{z}_1) - p_E(\underline{z}_2)) \frac{\underline{z}_1^{\sigma-1}}{\left(\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z\right)(\sigma-1)}\right],
\end{aligned} \tag{104}$$

which follows from substituting eq. (101) into eq. (99) and rearranging for E_3. Using eq. (104), we calculate

$$\frac{\partial}{\partial \underline{z}_2^{\sigma-1}} M_3 = E_1 \mu^E(\underline{z}_2) \left(\frac{\frac{\underline{z}_2^{\sigma-1} + \frac{1}{\sigma-1}\underline{z}_1^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} | z \geq \underline{z}_1]}}{1 + \frac{1}{\sigma-1}\frac{\underline{z}_1^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} | z \geq \underline{z}_1]}} - 1 \right). \tag{105}$$

If $\underline{z}_2 = \underline{z}_1$, one verifies that the term in parentheses is negative, while for large enough $\underline{z}_2$, it is positive. By continuity, there exists a $\underline{z}_2^*$, such that

$$\frac{\partial M_3}{\partial \underline{z}_2^{\sigma-1}} \begin{cases} < 0 \text{ if } \underline{z}_2 < \underline{z}_2^* \\ = 0 \text{ if } \underline{z}_2 = \underline{z}_2^* \\ > 0 \text{ if } \underline{z}_2 > \underline{z}_2^*. \end{cases} \tag{106}$$

Now, partial derivatives are

$$\frac{\partial [M_3/M_1]^{q-q^{CES}}}{\partial \underline{z}_2^{\sigma-1}} = (q - q^{CES}) \left[\frac{M_3}{M_1}\right]^{q-q^{CES}-1} \frac{1}{M_1} \frac{\partial M_3}{\partial \underline{z}_2^{\sigma-1}}, \tag{107}$$

$$\frac{\partial (L_3^p / L_1^p)}{\partial \underline{z}_2^{\sigma-1}} = \frac{-f_l^c}{L_1^p} \frac{\partial M_3}{\partial \underline{z}_2^{\sigma-1}}, \tag{108}$$

$$\begin{aligned}
\frac{\partial}{\partial \underline{z}_2^{\sigma-1}} \left[\frac{\int z^{\sigma-1} m_3(z)\,\mathrm{d}z}{\int z^{\sigma-1} m_1(z)\,\mathrm{d}z}\right]^{1/(\sigma-1)} &= \left[\frac{\int z^{\sigma-1} m_3(z)\,\mathrm{d}z}{\int z^{\sigma-1} m_1(z)\,\mathrm{d}z}\right]^{1/(\sigma-1)-1} \\
&\times \frac{1}{\sigma-1} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1}\mu^E(z)\,\mathrm{d}z} \frac{-1}{E_1(\sigma-1)} \frac{\partial M_3}{\partial \underline{z}_2^{\sigma-1}},
\end{aligned} \tag{109}$$

from which we see that eq. (108, 109) have the reverse sign order of (106), while eq. (107) has the same sign order, iff $q > q^{CES}$. Combined with the facts that each factor is 1 at f_l^c (no crisis in phase 2) and at $f_h^c \to \infty$, the graphs in panels A and B of Figure A.2 follow. Furthermore, if $q \leq q^{CES}$, then the graph for q' in panel C follows, and $(Y_3/Y_1)(q, f_h^c) \geq 1$ for all $f_h^c \geq f_l^c$.

Assume now that $q > q^{CES}$. For reference, note that

$$\frac{\int z^{\sigma-1} m_1(z)\ \mathrm{d}z}{\int z^{\sigma-1} m_3(z)\ \mathrm{d}z} = 1 + \frac{M_1 - M_3}{E_1(\sigma-1)} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z)\ \mathrm{d}z} \tag{110}$$

is the ratio of productivity integrals. The output ratio only depends on f_h^c through $\underline{z}_2$, and it depends on $\underline{z}_2$ only through M_3. Consider the output ratio now as a function of M_3 given q, i.e. $\frac{Y_3}{Y_1}(M_3; q)$, and note that the domain of $M_3(\underline{z}_2)$ is $[M_{min}, M_1]$ for some M_{min}, and that M_3 runs from M_1 monotonically down to $M_{min} := M_3(\underline{z}_2^*)$ and back up to M_1 in the limit. We write

$$\frac{Y_3}{Y_1}(M_3; q) = F_1(M_3, q) F_2(M_3) F_3(M_3),$$

corresponding to its three contributing factors, and take the derivative

$$\frac{\partial}{\partial M_3} \frac{Y_3}{Y_1} = (F_1'(M_3, q)/F_1(M_3, q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3)) \frac{Y_3}{Y_1}(M_3; q) \tag{111}$$

$$=: g(M_3, q) \frac{Y_3}{Y_1}(M_3; q). \tag{112}$$

The derivative vanishes iff (substituting in)

$$0 = (F_1'(M_3, q)/F_1(M_3, q) + F_2'(M_3)/F_2(M_3) + F_3'(M_3)/F_3(M_3)) \tag{113}$$

$$= \frac{(q - q^{CES})}{M_3} \underbrace{- \frac{f_l^c}{L_3^p(M_3)} - \frac{\int z^{\sigma-1} m_1(z)\ \mathrm{d}z}{\int z^{\sigma-1} m_3(z)\ \mathrm{d}z} \frac{1}{\sigma - 1} \frac{\underline{z}_1^{\sigma-1}}{\int_{\{z \geq \underline{z}_1\}} z^{\sigma-1} \mu^E(z)\ \mathrm{d}z} \frac{1}{E_1(\sigma-1)}}_{:= -s(M_3)/M_3} \tag{114}$$

$$= v(M_3, q), \tag{115}$$

which is strictly decreasing in M_3 (to see that the third summand is decreasing cf. eq. 110). Therefore, if M_3^* satisfying eq. (113) exists, it is unique. Since $M_{min} \leq M_3 \leq M_1$, it can only exist if $v(M_{min}, q) \geq 0 \geq v(M_1, q)$. Clearly, if q is large enough, then v must be globally positive and if q gets close to q^{CES}, then v is negative for some, and eventually all $M_3 \in [M_{min}, M_1]$. One checks that an interior critical point exists iff $q \in (\underline{q}, \overline{q})$, where $\underline{q} = q^{CES} + s(M_1)$ and $\overline{q} = q^{CES} + s(M_{min})$. On the one hand, if $q \leq \underline{q}$, even though there is LOV relative to CES, the output ratio is decreasing in the number of post-crisis varieties, and any crisis is cleansing. On the other hand, if $q \geq \overline{q}$, then the output ratio is globally increasing in the number of varieties, and all crises come at an output loss. For the region in between, $q \in (\underline{q}, \overline{q})$, there is an extremum at some M_3^*. This is a maximum because Y_3/Y_1 satisfies second-order conditions at M_3^* for such q. See this as follows. As argued, $g(M_3; q)$ is decreasing, so $g' < 0$. The second derivative of Y_1/Y_3 evaluated at M_3^* is

$$g'(M_3^*; q) \frac{Y_3}{Y_1}(M_3^*; q) + g(M_3^*; q) \frac{Y_3}{Y_1}'(M_3^*; q) = g'(M_3^*; q) \frac{Y_3}{Y_1}(M_3^*; q) + g(M_3^*; q) \times 0 < 0. \tag{116}$$

Consequently, there is an optimal number of firms in phase 3, $M_3^* \in (M_{min}, M_1)$, created by an "optimally cleansing business cycle". Because the mapping between f_h^c and M_3 is parabola-shaped, its inverse image always has two elements, and any optimal post-crisis number of firms can be created by two different values of f_h^c. This implies the graph displayed in panel C for q''. It is easy to check that the output ratio uniformly decreases in q, which explains why the graph for q''' lies below. Therefore, as the curve of Y_3/Y_1 shifts down, there must be an interval $(q_\circ, q^\circ) \subseteq (\underline{q}, \overline{q})$ where the output ratio crosses 1 multiple times, creating two nonempty, disjunct intervals in which crises are cleansing.

The partial equilibrium result follows immediately since F_2 and F_3 are independent of f_h^c and of q, by Proposition 1. ■

Proof Proposition 6. We start by characterizing the optimal policy in the case of entry and then study the one of pure exit. For lighter notation, we suppress the time subscripts of (22). Accordingly, let $m^I = m_{t-1}$ be the density of incumbents, and $I = \int m^I$ the incumbent mass.

Entry: The interior first-order condition on entry is

$$E^{SP}(\underline{z}; m^I, q) = \frac{\bar{L} - If^c}{p_E(\underline{z})f^c + f^e}\frac{1}{\sigma} - \frac{\int_{\{z \geq \underline{z}\}} z^{\sigma-1} m^I \, \mathrm{d}z}{\int_{\{z \geq \underline{z}\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z}\frac{\sigma - 1}{\sigma} + \Delta E^{SP}, \tag{117}$$

which equals the CES GE-entry plus a difference term, ΔE^{SP}. The latter is given by

$$\Delta E^{SP} = \left(q - \frac{1}{\sigma - 1}\right) L^p \frac{\sigma}{\sigma - 1}(p_E(\underline{z})f^c + f^e)^{-1} Q(m^I, \underline{z}), \tag{118}$$

where $Q(m^I, \underline{z})$ is the ratio between the average productivity of firms post-entry and the average productivity of (successful) entrants. It is immediate that if the household has CES preferences ($q = 1/(\sigma - 1)$), then $\Delta E^{SP} = 0$, and the market outcome is constrained efficient. Over time, a stock of firms builds up (see discussion in Section 3.1), so $\sum_{t \geq 0} E_t^{SP} \to E^{SP,ss}$, where we call $E^{SP,ss}$ the (steady-state) total mass of entrants for the social planner problem. As E_t^{SP} tends to 0, Q tends to 1, and the incumbent mass building up is, eventually, $\widetilde{I} = I + E^{SP,ss}$. (Similarly, the measure of incumbents is stacked up.) Substituting these into eq. (117) and rearranging, we find

$$E^{SP,ss} = \frac{(\bar{L} - If^c)q}{f^c p_E(\underline{z})(1 + q) + f^e} - \frac{\int_{\{z \geq \underline{z}\}} z^{\sigma-1} m^I \, \mathrm{d}z}{\int_{\{z \geq \underline{z}\}} z^{\sigma-1} \mu^E(z) \, \mathrm{d}z} \frac{f^c p_E(\underline{z}) + f^e}{f^c p_E(\underline{z})(1 + q) + f^e}. \tag{119}$$

The socially optimal number of entrants is increasing in taste-for-varieties, q, and decreasing in the mass and productivity of active incumbents. Fixed costs of entry, f^e, stop to matter if and only if $q \to \infty$. Comparing this to eq. (15), it so follows that the general equilibrium steady-state mass converges to $E^{SP,ss}$ if and only if $q = \frac{1}{\sigma-1}$. It also follows that in an economy with high $q > \frac{1}{\sigma-1}$, the equilibrium number of entrants is too small compared to the short-run optimal number, so $E^{GE} < E^{SP}$. Conversely, for small q, the equilibrium number of varieties is larger than the short-run efficient number.

Using the first order condition on E^{SP}, the interior first order condition on $\underline{z}^{SP} = \underline{z}^{SP}(E; q, m^I)$ yields:

$$(\bar{L} - If^c)\left(\int_{\{z \geq \underline{z}\}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z\right)$$
$$\left(\frac{f^e}{f^c p_E(\underline{z}) + f^e}\frac{1}{\sigma}\frac{J}{K} - \left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right]\frac{1}{\sigma}\frac{\sigma - \frac{J}{K}}{\sigma - 1}\right)$$
$$= \left(\int_{\{z \geq \underline{z}\}} z^{\sigma-1} m^I\ \mathrm{d}z\right)(f^c p_E(\underline{z}) + f^e)$$
$$\left(\frac{f^e}{f^c p_E(\underline{z}) + f^e}\left(1 - \frac{\sigma-1}{\sigma}\frac{1}{K}\right) + \frac{1}{\sigma}\frac{1}{K}\left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right]\right). \tag{120}$$

The fraction J/K and J and K individually take value 1 precisely when $q = \frac{1}{\sigma-1}$. Only in this case, are the terms in the large parentheses both equivalent to $\frac{f^e \int_{\{z \geq \underline{z}\}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z}{\delta_f} - \left(\int_{\{z \geq \underline{z}\}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z - \underline{z}^{\sigma-1}p_E(\underline{z})\right)$. Eq. (120) is then equivalent to eq. (7), the GE cutoff.

In general, we need to unpack J and K to see how the cutoff is determined in a non-CES world. Here, note that $J = 1 + [(\sigma-1)q - 1]Q$ and $K = 1 + \frac{1}{\sigma}[(\sigma-1)q - 1]Q$. If we approach a steady state (or drop the incumbents), $Q \to 1$. Then eq. (120) simplifies:

$$\frac{f^e}{f^c p_E(\underline{z}) + f^e}(\sigma - 1)q - \left[1 - \frac{\underline{z}^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right] = 0, \tag{121}$$

and under mild regularity conditions on μ^E, we can conclude that $\frac{\partial \underline{z}}{\partial q} < 0$. Intuitively, if the economy has a strong preference for varieties, the entry barriers for new establishments should be lower. Eq. (121) also holds with entrants, if all entrants were drawn from the same distribution, i.e., if $m^I \propto \mu^E(z)\mathbb{I}(z \geq \underline{z})$.

Exit: Suppose incumbent firms are distributed according to $m^I = M_0\mu^E(z)\mathbb{I}(z \geq \underline{z}_0)$, and that fixed costs f^c are such that the optimal cutoff $\underline{z}$ is larger than $\underline{z}_0$, and that optimal entry, therefore, is nil. Hence, the relevant production function to maximize is

$$Y = (M_0)^q\, p_E(\underline{z})^{q - \frac{1}{\sigma-1}}\left(\bar{L} - f^c(M_0 p_E(\underline{z}))\right)\left(\int_{\{z \geq \underline{z}\}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z\right)^{\frac{1}{\sigma-1}}, \tag{122}$$

where taking first order conditions yields

$$f^c M_0 p_E(\underline{z}^{SP}) = \frac{\bar{L} - f^c M_0 p_E(\underline{z}^{SP})}{\sigma - 1}\left([q(\sigma-1) - 1] + \frac{(\underline{z}^{SP})^{\sigma-1}}{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{SP}]}\right), \tag{123}$$

which is equal to the market equilibrium outcome only if $q = \frac{1}{\sigma-1}$. For larger q, the cutoff is again optimally set lower than in the equilibrium allocation. ∎

Proof of Proposition 7. In the steady-state targeted by the social planner, eq. (123) must hold once entry has subsided, i.e., with $E^{SP} = 0$ and some $\underline{z}^{SP}$. We now evaluate eq. (5) at this $\underline{z}^{SP}$, whereby we have swapped f^c for $\delta^c f^c$ beforehand. Solving for δ^c yields the result. ■

Proof of Lemma 3. Since we are using the entry equation for positive entry, all derivatives are right-derivatives. Let E^I be the number of 'old' entrants. That is, the mass of incumbent firms that have tried, and of which $E^I p_E(z^*)$ have succeeded to enter the market. By definition, $M = Ep_E(\underline{z}) + E^I p_E(z^*)$. The entry condition determines $E = E(\mathcal{I})$ such that

$$Ep_E(\underline{z}) + E^I p_E(z^*) = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]} - E^I p_E(z^*) \left(1 - \frac{\mathbb{E}[z^{\sigma-1} \mid z \geq z^*]}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right). \tag{124}$$

Letting

$$k(\underline{z}, z^*) = p_E(z^*) \left(1 - \frac{\mathbb{E}[z^{\sigma-1} \mid z \geq z^*]}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}]}\right), \tag{125}$$

we obtain

$$k(\underline{z}, z^*) \begin{cases} > 0 & \text{for } \underline{z} > z^*, \\ = 0 & \text{for } \underline{z} = z^*, \\ < 0 & \text{for } \underline{z} < z^*. \end{cases} \tag{126}$$

One now takes elasticities with respect to $\mathcal{I}$ to obtain the expression for $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}$. Next, substitute the distribution post-entry

$$\frac{Ep_E(\underline{z})\mu^E(z \mid z \geq \underline{z}) + E^I p_E(z^*)\mu^E(z \mid z \geq z^*)}{Ep_E(\underline{z}) + E^I p_E(z^*)} \tag{127}$$

into the integral of the average productivity, $\log \bar{z}$. After some algebra to write integrals as expectations, substitute out $E(\mathcal{I})$ to obtain

$$\log \bar{z} = \frac{1}{\sigma - 1} \left(\log \mathcal{I} - \log \sigma f^c + \log \underline{z} - \log[p_E(\underline{z}) E_{|E^I=0} + E^I k(\underline{z}, z^*)]\right) \tag{128}$$

$$= \frac{1}{\sigma - 1} (\log \mathcal{I} - \log \sigma f^c + \log \underline{z} - \log M) \tag{129}$$

with

$$E_{|E^I=0} = \frac{\mathcal{I}}{\sigma f^c} \frac{\underline{z}^{\sigma-1}}{\mathbb{E}[z^{\sigma-1} \mid z \geq \underline{z}] p_E(\underline{z})}. \tag{130}$$

Differentiating with respect to $\log \mathcal{I}$ yields the result. ■

Proof of Proposition 8. From Lemma 3, $k(\underline{z}, \underline{z}) = 0$. Hence, $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} = 1$ and $\frac{\partial_+ \log \bar{z}}{\partial_+ \log \mathcal{I}} = 0$. ■

Proof of Remark 5. We prove the general statement, and the special case of constant returns to scale (homogeneity of degree 1) follows. First, we prove that if output changes have a representation like eq. (24) under homogeneity of degree χ, then we identify $\chi+q$. Then we prove that such a representation exists.

First, we prove the proportionality of input demands. Consider a production function $f(\mathbf{x})$ homogeneous of degree χ, increasing, strictly concave, and twice differentiable in all inputs. Then, we can write generic implicit input demands as $f_{x_i}=w_i$, where w_i is the unit price of input x_i. Taking ratios of the FOCs $f_{x_i}/f_{x_j}=\psi(x_i/x_j)=w_i/w_j$, which we can invert to obtain $x_i=\phi(w_i/w_j)x_j$, $\forall i,j$. This proves that all inputs are perfectly collinear, provided that relative input prices do not change. To prove the second part of the statement, we substitute input demands into the production function, $f(x_1,...,x_N)=(\phi_{1j}x_j,...,\phi_{Nj}x_j)$. By homogeneity of degree χ, we obtain $f(x_1,...,x_N)=x_j^\chi f(\phi_{1j},...,\phi_{Nj})$. This directly implies that the output elasticity to a single input is

$$\frac{\partial\log y}{\partial\log x_j}=\chi.$$

This proves that if we can write output changes in an equivalent way to eq. (24) for general homogeneous production functions, the regression estimates $\widehat{\beta}=\chi+q$. To prove that such a representation exists, note that by the properties of homogeneous functions, the marginal cost associated to the cost minimizing input bundle takes the form $c(z)=\frac{\kappa}{z\chi}\left(\frac{y(z)}{z}\right)^{\frac{1}{\chi}-1}$, where κ is some function of the relative input prices and parameters which the firm takes as given. By the profit maximization, then the firm charges an optimal price $p(z)=\frac{\sigma}{\sigma-1}c(z)$. Using the demand function and solving for profits we obtain $\pi(z,m)=\frac{\mathcal{I}}{\sigma}\frac{c(z)^{1-\sigma}}{\int_z c(z)^{1-\sigma}m(z)\ \mathrm{d}z}-f^c$. From the demand function we can solve output and find that $y(z)\propto z^{\frac{\sigma}{\chi+\sigma(1-\chi)}}$. From this, the steps used in the proof of Lemma 3 apply since we can write profits as $\pi(z,m)=\frac{\mathcal{I}}{\sigma}\frac{z^{f(\sigma,\chi)}}{\int_z z^{f(\sigma,\chi)}m(z)\ \mathrm{d}z}-f^c$, where $f(\sigma,\chi)$ is some constant that depends on the degree of homogeneity of the production function and the elasticity of substitution. ■

Proof of Remark 6. Consider the methodology described in Appendix C.1 to obtain real output from nominal output and price indices in WIOD. Define sales $S_t=P_tY_t$. The data either reports sales at previous year prices correctly accounting for the variety effect $S_t^{t-1}=P_{t-1}Y_t$ or sales at previous year prices using a deflator that ignores the variety effect $\widetilde{S}_t^{t-1}=\frac{S_t^tP_{t-1}^{CES}}{P_t^{CES}}$, with $P_t^{CES}=P_tM_t^{q-\frac{1}{\sigma-1}}$. When the statistical agencies report deflators that properly account for the variety effect, then real output growth can be decomposed as:

$$\begin{aligned}\Delta\log Y_t&=\log S_t^{t-1}-\log S_{t-1}^{t-1}=\log P_{t-1}+\log Y_t-\log P_{t-1}-\log Y_{t-1}\\&=q\Delta\log M_t+\Delta\log L_t^p+\Delta\log\bar{z}_t.\end{aligned}\tag{131}$$

When the statistical agencies ignore the variety effect, then real output growth can be decomposed as:

$$\Delta\log\widetilde{Y}_t=\log\widetilde{S}_t^{t-1}-\log S_{t-1}^{t-1}=\log P_t+\log Y_t+\log P_{t-1}^{CES}-\log P_t^{CES}-\log P_{t-1}-\log Y_{t-1}$$

$$
\begin{aligned}
&= \log Y_t + \left(q - \frac{1}{\sigma - 1}\right) \log M_{t-1} - \left(q - \frac{1}{\sigma - 1}\right) \log M_t - \log Y_{t-1} \\
&= q \Delta \log M_t + \Delta \log L_t^p + \Delta \log \bar{z}_t - \left(q - \frac{1}{\sigma - 1}\right) \Delta \log M_t \\
&= \frac{1}{\sigma - 1} \Delta \log M_t + \Delta \log L_t^p + \Delta \log \bar{z}_t. \qquad (132)
\end{aligned}
$$

Let the fixed-cost technology be Cobb–Douglas in the final good and labor:

$$
B(Y, L) \;=\; \kappa(\alpha)\, Y^\alpha L^{1-\alpha}, \qquad \kappa(\alpha) = \alpha^\alpha (1 - \alpha)^{1-\alpha}, \quad \alpha \in [0, 1].
$$

Let (P_t, w_t) denote the prices of the final good and labor, then the cost of the bundle B is

$$
\Phi_t(P_t, w_t) \;=\; P_t^\alpha w_t^{1-\alpha}.
$$

The firm requires f^c units of B per period. With labor as the numeraire ($w_t = 1$), the nominal expenditure on fixed cost payments at time t is

$$
f^c \cdot \Phi_t(P_t, 1) \;=\; f^c\, P_t^\alpha.
$$

This nests two special cases. If $\alpha = 0$, then the fixed costs are paid only in units of labor, and the nominal expenditure is $w_t f^c = f^c$. If $\alpha = 1$, then the fixed costs are paid only in final-good units, and the nominal expenditure is $f^c P_t$.

Let the economy be in a state of entry, in line with our estimation strategy that isolates periods of positive demand shocks. Then, the mass of varieties in the economy is

$$
\begin{aligned}
&\frac{L_t^p\ \underline{z}_t^{\sigma-1}}{(\sigma - 1)\ P_t^\alpha\ f^c} = \mathcal{Z}_t^{\sigma-1} \\
&\frac{L_t^p}{\sigma^\alpha (\sigma - 1)^{1-\alpha}} M_t^{\alpha\left(q - \frac{1}{\sigma-1}\right)} \mathcal{Z}_t^{\alpha+1-\sigma} \underline{z}_t^{\sigma-1} = f^c \\
&\frac{L_t^p \underline{z}_t^{\sigma-1}}{\sigma^\alpha (\sigma - 1)^{1-\alpha}} M_t^{\alpha\left(q - \frac{1}{\sigma-1}\right) + \frac{\alpha+1-\sigma}{\sigma-1}} \bar{z}_t^{\alpha+1-\sigma} = f^c \\
&M_t = \left(\frac{\sigma^\alpha (\sigma - 1)^{1-\alpha} f^c}{L_t^p \underline{z}_t^{\sigma-1}} \bar{z}_t^{\sigma-1-\alpha} \right)^{\frac{1}{\alpha q - 1}} \qquad (133)
\end{aligned}
$$

As $\alpha \to 1$, we obtain the mass of varieties in an economy where fixed costs are paid exclusively in final-good units, that is

$$
M_t = \left(\frac{\sigma f^c}{L_t^p \underline{z}_t^{\sigma-1}} \bar{z}_t^{\sigma-2} \right)^{\frac{1}{q-1}}.
$$

As $\alpha \to 0$, we obtain the mass of varieties in an economy where fixed costs are paid exclusively in units of labor, that is

$$
M_t = \frac{L_t^p}{(\sigma - 1) f^c} \frac{\underline{z}_t^{\sigma-1}}{\bar{z}_t^{\sigma-1}} \iff f^c = \frac{L_t^p}{\sigma - 1} \frac{\underline{z}_t^{\sigma-1}}{\int z^{\sigma-1} m_t(z) dz}.
$$

Taking log differences of eq. (133), we obtain

$$\Delta \log M_t = \frac{1}{\alpha q - 1}\left[-\Delta \log L_t^p - (\sigma-1)\Delta \log \underline{z}_t + (\sigma - 1 - \alpha)\Delta \log \bar{z}_t\right]. \tag{134}$$

Substituting $\Delta \log M_t$, eq. (134), into $\Delta \log Y_t$ and $\Delta \log \widetilde{Y}_t$, eq. (131) and eq. (132), we obtain

$$\Delta \log Y_t = \left(1 - \frac{q}{\alpha q - 1}\right)\Delta \log L_t^p - \frac{q(\sigma-1)}{\alpha q - 1}\Delta \log \underline{z}_t + \frac{q(\sigma-1)-1}{\alpha q - 1}\Delta \log \bar{z}_t,$$
$$\Delta \log \widetilde{Y}_t = \left(1 - \frac{1}{(\sigma-1)(\alpha q - 1)}\right)\Delta \log L_t^p - \frac{1}{\alpha q - 1}\Delta \log \underline{z}_t + \frac{\alpha q(\sigma-1)-\alpha}{(\sigma-1)(\alpha q - 1)}\Delta \log \bar{z}_t.$$

Given that $L_t^p = \frac{\sigma-1}{\sigma}\mathcal{I}_t$, the above equations hold for both $\mathcal{I}_t$ and L_t^p. Furthermore, positive income shocks do not move $\underline{z}_t$, and by assumption 2 they also do not move $\bar{z}_t$. More specifically, $\Delta \log X_t > 0 \implies \Delta \log \underline{z}_t = 0$ and $\Delta \log \bar{z}_t = 0$, where $X_t \in \{\mathcal{I}_t, L_t^p\}$. Substituting these conditions and indexing by industry i yields the regression specifications in the Remark

$$\Delta \log Y_{i,t} = \beta \Delta \log X_{i,t} + \epsilon_{i,t},$$
$$\Delta \log \widetilde{Y}_{i,t} = \widetilde{\beta} \Delta \log X_{i,t} + \epsilon_{i,t},$$

with $\beta = 1 - \frac{q}{\alpha q - 1}$, $\widetilde{\beta} = 1 - \frac{1}{(\sigma-1)(\alpha q-1)}$.

Using the relationships $\beta = 1 - \frac{q}{\alpha q - 1}$ and $\widetilde{\beta} = 1 - \frac{1}{(\sigma-1)(\alpha q-1)}$, we invert them to obtain, as in the Remark,

$$q(\alpha) = \frac{1-\beta}{\alpha(1-\beta)-1},$$
$$\widetilde{q}(\alpha) = \frac{1}{\alpha}\left[1 + \frac{1}{(\sigma-1)(1-\widetilde{\beta})}\right],$$

where $\widetilde{q}(\alpha)$ denotes the function that maps α (given $\widetilde{\beta}$ and σ) to the implied q under incorrectly measured (non–variety-adjusted) deflators, in contrast to $q(\alpha)$ obtained under variety-adjusted deflators.

Identification is immediate from these mappings: with variety-adjusted deflators, q is point-identified for any $\alpha \in [0,1]$; with deflators that ignore variety, q is point-identified for any $\alpha \in (0,1]$ (given σ), but *not* at $\alpha = 0$, where the mapping is undefined and $\lim_{\alpha \downarrow 0} \widetilde{q}(\alpha) = +\infty$.

Applying Proposition 8, the corresponding biases can be computed as follows (the biases arise because the economy has a single true, but unobserved, α). For the case of deflators which correctly account for the variety effect, the bias is given by

$$\text{Bias}(\alpha;\beta) = \underbrace{(\beta - 1)}_{\text{measured q}} - \underbrace{\frac{1-\beta}{\alpha(1-\beta)-1}}_{\text{true } q} = -\frac{\alpha(1-\beta)^2}{\alpha(1-\beta)-1}.$$

For the case of deflators which ignore the variety effect, the bias is given by

$$\begin{aligned}\text{Bias}(\alpha;\widetilde{\beta},\sigma) &= \underbrace{(\widetilde{\beta}-1)}_{\text{measured q}} - \underbrace{\frac{1}{\alpha}\left[1+\frac{1}{(\sigma-1)\big(1-\widetilde{\beta}\big)}\right]}_{\text{true } q} \\ &= \frac{\alpha(\sigma-1)(\widetilde{\beta}-1)^2-(\sigma-1)(\widetilde{\beta}-1)+1}{\alpha(\sigma-1)(\widetilde{\beta}-1)}.\end{aligned}$$

■

Proof of Proposition 9. Using Lemma 3, note that

$$\frac{\partial_+ \log Y}{\partial_+ \log \mathcal{I}} = q\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} + 1 + \frac{1}{\sigma-1}\left(1-\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}\right) \tag{135}$$

The LHS is the estimated coefficient from our regression. Define the implied $\widehat{q} = \beta - 1$. Then, we have

$$\widehat{q} = \beta - 1 = \frac{1}{\sigma-1} + \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}(q - q^{CES}) \tag{136}$$

If we have a measure of $\frac{1}{\sigma-1}$ we can write

$$\widehat{q} - \frac{1}{\sigma-1} = \beta - 1 - \frac{1}{\sigma-1} = \frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}}(q - q^{CES}). \tag{137}$$

By Lemma 3, $k(\underline{z}, z^\star) < 0$ if $z^\star > \underline{z}$, which implies $\frac{\partial_+ \log M}{\partial_+ \log \mathcal{I}} > 1$, and hence we have that

$$\text{sgn}\left(\widehat{q} - \frac{1}{\sigma-1}\right) = \text{sgn}\left(q - \frac{1}{\sigma-1}\right).$$

■

Proof of Proposition A.1. By Matsuyama and Ushchev (2023), the only demand system in HSA, HIIA or HDIA that does not feature an aggregate externality to TFP and has constant LoV is CES. Since $\frac{d \ln Y^{CES}}{d \ln M} = \frac{1}{\sigma-1}$, we immediately get $g(\boldsymbol{y}) = Y^{CES}(\boldsymbol{y})M^{\frac{-1}{\sigma-1}}$ has LoV of 0. By iso-elasticity of h, we get $h(M) = M^q$. ■

Proof of Proposition A.2. Let μ^E be the baseline probability density function from which productivity is drawn. Let m^I be a density of incumbents (in a dynamic context, $m^I = m_{t-1}$ at time t), and $I = \int m^I$ the incumbent mass. We then have the following iterative entry procedure:

1. At iteration $k \geq 0$, a mass of firms, $M^{(k)}$, enter initially until:

$$\begin{aligned}0 &= \mathbb{E}_{\mu^E}[\max\{\pi(z, m^I, m^{(k-1)}, M^{(k)}), 0\}] - f^e \\ &= \int_{\underline{z}^{(k)}}^{\infty}\left[\frac{\mathcal{I}}{\sigma}\frac{z^{\sigma-1}}{\int z^{\sigma-1}m' \, \mathrm{d}z + \int z^{\sigma-1}m^I \, \mathrm{d}z} - f^c\right]\mu^E \, \mathrm{d}z - f^e\end{aligned} \tag{138}$$

where $m' := M^{(0)}\mu^E$ in the first iteration and $m' := m^{(k-1)} + M^{(k)}\mu^E$ in the kth iteration.

2. Currently active firms check their profitability, and all firms with productivity less than $\underline{z}^{(k)}$ leave the market. The cut-off $\underline{z}^{(k)}$ is determined by:

$$\pi(\underline{z}^{(k)}, m^I, m^{(k-1)}, M^{(k)}) = \frac{\mathcal{I}}{\sigma} \frac{(\underline{z}^{(k)})^{\sigma-1}}{\int z^{\sigma-1} m' \,\mathrm{d}z + \int z^{\sigma-1} m^I \,\mathrm{d}z} - f^c = 0$$
$$(\underline{z}^{(k)})^{\sigma-1} = f^c \frac{\sigma}{\mathcal{I}} \left[\int z^{\sigma-1} m' \,\mathrm{d}z + \int z^{\sigma-1} m^I \,\mathrm{d}z \right]$$

3. After exit, the density function corresponding to the firms left in the market is relabeled as the new incumbent density function and is given by:

$$m^I \leftarrow m^I + m'(z)\mathbb{I}_{\{z \geq \underline{z}^{(k)}\}} = m^I + m^{(k)}$$

where m^I is not affected by the truncation since scope for entry implies that the threshold is weakly lower than the one previously incurred by the incumbents.

Steps 1 to 3 are iterated until convergence.

Equivalence of threshold $\underline{z}$

By step 1, we have that at iteration k the following holds:

$$f^e = \int_{\underline{z}^{(k)}}^{\infty} \left[\frac{\mathcal{I}}{\sigma} \frac{z^{\sigma-1}}{\int z^{\sigma-1} m^{(k-1)} \,\mathrm{d}z + M^{(k)} \int z^{\sigma-1} \mu^E \,\mathrm{d}z + \int z^{\sigma-1} m^I \,\mathrm{d}z} - f_c \right] \mu^E \,\mathrm{d}z \tag{139}$$

$$= \frac{\mathcal{I}}{\sigma} \frac{p_E(\underline{z}^{(k)}) \, \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{\int z^{\sigma-1} m^{(k-1)} \,\mathrm{d}z + M^{(k)} \mathbb{E}_{\mu^E}[z^{\sigma-1}] + \int z^{\sigma-1} m^I \,\mathrm{d}z} - p_E(\underline{z}^{(k)}) f_c \tag{140}$$

where $p_E(\underline{z}^{(k)}) = \int_{\underline{z}^{(k)}}^{\infty} \mu^E(z) \,\mathrm{d}z$.
Solving for $M^{(k)}$, we obtain:

$$M^{(k)} = \frac{1}{\mathbb{E}_{\mu^E}[z^{\sigma-1}]} \left[\frac{\mathcal{I}}{\sigma} \frac{p_E(\underline{z}^{(k)}) \, \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c p_E(\underline{z}^{(k)})} - \int z^{\sigma-1} m^{(k-1)} \,\mathrm{d}z - \int z^{\sigma-1} m^I \,\mathrm{d}z \right] \tag{141}$$

Substituting the derived $M^{(k)}$ in the cut-off equation of step 2, we obtain:

$$(\underline{z}^{(k)})^{\sigma-1} = f^c \frac{p_E(\underline{z}^{(k)}) \, \mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{f^e + f^c \, p_E(\underline{z}^{(k)})} \tag{142}$$

$$\frac{f^e}{f^c} = p_E(\underline{z}^{(k)}) \left[\frac{\mathbb{E}_{\mu^E}[z^{\sigma-1} \mid z \geq \underline{z}^{(k)}]}{(\underline{z}^{(k)})^{\sigma-1}} - 1 \right] \tag{143}$$

First, note that clearly none of this is dependent on having started at iteration k. We would have obtained eq. (143) also by having derived $M^{(0)}$ in eq. (141). Second, note that $\underline{z}^{(k)}$ is a constant sequence as it depends only on f^c and f^e, which do not change with k. Hence, we can write $\underline{z}_k = \underline{z}\ \forall k \in \mathbb{N}$. Third, note that the equation pinning down the threshold in the iterative entry procedure is the same as in the simultaneous entry game eq. (7).

Equivalence of entry mass E

With $\underline{z}^{(k)} = \underline{z}\ \forall k \in \mathbb{N}$, it is clear that all we are doing is stacking scaled versions of the baseline density truncated at $\underline{z}$. Therefore, eq. (141) can be written as:

$$M^{(k)} = \frac{1}{\mathbb{E}_{\mu^E}[z^{\sigma-1}]}\left[\frac{\mathcal{I}}{\sigma}\frac{\underline{z}^{\sigma-1}}{f^c} - (M^{(0)} + ... + M^{(k-1)})\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E \,\mathrm{d}z - \int z^{\sigma-1} m^I \,\mathrm{d}z\right] \tag{144}$$

where we used eq. (142) to substitute in $\frac{\underline{z}^{\sigma-1}}{f^c}$.
This shows that $\{M^{(k)}\}$ is a decreasing sequence. Since $M^{(k)} \geq 0$ holds $\forall k$, $\{M^{(k)}\}$ has a real limit point, and this limit point must be 0. Hence, taking the limit as $k \to \infty$, we obtain:

$$0 = \frac{\mathcal{I}}{\sigma}\frac{\underline{z}^{\sigma-1}}{f^c} - \sum_{i=0}^{\infty} M^{(i)} \int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E \,\mathrm{d}z - \int z^{\sigma-1} m^I \,\mathrm{d}z \tag{145}$$

$$\sum_{i=0}^{\infty} M^{(i)} = E = \frac{\mathcal{I}}{\sigma f^c}\frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E \,\mathrm{d}z} - \frac{\int z^{\sigma-1} m^I \,\mathrm{d}z}{\int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E \,\mathrm{d}z} \tag{146}$$

which is clearly the same entry mass as in the simultaneous entry game (see eq. 6).

Equivalence of measure μ

Assuming that μ^E is a bounded density function, it is clear that $m^{(k)} := \sum_{i=0}^{k} M^{(i)}\mu^E(z)\mathbb{I}_{\{z\geq\underline{z}\}}$ converges to $m^E := E\mu^E(z)\mathbb{I}_{\{z\geq\underline{z}\}}$ as $k \to \infty$, given that the partial sums of the entry mass series form a Cauchy sequence. The limiting density function $m = m^I + E\mu^E(z)\mathbb{I}_{\{z\geq\underline{z}\}}$ is the same density function as in the simultaneous entry game equilibrium. ■

Proof of Proposition A.3. We prove the first statement by proving that factor-saving effects are dampened by the endogenous factor supply response. We prove it for a single primary factor with the understanding that it readily generalizes.

Suppose the household has preferences

$$\mathcal{U} = \log C - \nu(L) \tag{147}$$

and BC

$$wL + \Pi = PC \tag{148}$$

Setting labor as the numeraire, we have that the labor supply curve is implicitly defined by

$$\nu'(L) = (PC)^{-1} \tag{149}$$

Assuming that $\nu(\cdot)$ is twice continuously differentiable, we can write the labor market clearing as

$$L^p + Mf_c + Ef_e = L = \nu'^{-1}((PC)^{-1}) \tag{150}$$

From the price index and consumption aggregators, we know that $PC = L^p\mu$, which implies

$$Mf_c + Ef_e = \nu'^{-1}((L^p\mu)^{-1}) - L^p \tag{151}$$

Note that RHS is decreasing in L^p for all $\nu' > 0$ and $\mu > 1$. So, more firms (M) or more entrants (E) reduce labor used in production, even when supply is elastic. To complete the proof, define appropriately a function g, with $g' < 0$ such that

$$Mf_c + Ef_e = g(L^p) - L^p \tag{152}$$

Comparing it with the inelastic case

$$Mf_c + Ef_e = \bar{L} - L^p \tag{153}$$

we note that for any given change of the LHS ΔLHS under inelastic supply, $\Delta L^p = -\Delta LHS$. Under elastic supply, if $\Delta L^p = -\Delta LHS$ then $|\Delta RHS| > |\Delta LHS|$. We conclude that under elastic labor supply, for any given ΔLHS we need $|\Delta L^p| < |\Delta LHS|$. Hence, elastic labor dampens movements in production labor. Since the elastic factor supply dampens movements in factors used in production, it reduces the factor-saving effects. As a consequence, all else equal, the output change is strictly smaller under elastic factor supply.

■

Proof of Remark A.1 and Proposition A.4. Since firms are selected on z, the average externality contributions in the economy are given by

$$\mathbb{E}_{m_1}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_1], \tag{154}$$

$$\mathbb{E}_{m_2}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_2], \tag{155}$$

$$\mathbb{E}_{m_3}[\xi] = \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_2] \times \frac{M_2}{M_3} + \mathbb{E}_{\mu^E}[\xi \mid z \geq \underline{z}_1] \times \frac{E\mu_z(z \geq \underline{z}_1)}{M_3}. \tag{156}$$

where the first equation obtains from initial entry, the second from keeping only surviving firms, and the third from averaging the surviving firms of the cycle with new entrants. As usual, $E > 0$ is the post-cycle entry mass of firms. Let Δ be the difference operator differencing cycle phase 3 and 1. We immediately see that if externality and productivity draws are mean-independent, then $\mathbb{E}_{m_t}[\xi] = \mathbb{E}_{\mu^E}[\xi]$ $\forall t = 1, 2, 3$. Thus, $\mathcal{M}_t = M_t$, and the model is identical to our baseline model in aggregates. This proves Remark A.1.

Substituting into the aggregate externality $\mathcal{M}$, we obtain (46). This tells us that the welfare effect running through $\mathcal{M}$ is positive if and only if the selection in ξ given by $\Delta \log \mathbb{E}_m[\xi]$ is at least some positive number $-\Delta \log M$. This is a result analogous to Proposition 1. To relate the cleansing effect in this extension to the baseline model, and thus to complete the proof of Proposition A.4, compute $\Delta \log Y$ under the alternative aggregator. This yields

$$\Delta \log Y = q\ \Delta \log \mathbb{E}_m[\xi] + \underbrace{\left(q - \frac{1}{\sigma - 1}\right) \Delta \log M}_{\Delta Y^{hom}}. \tag{157}$$

One then notices that $\Delta \log \mathbb{E}_m[\xi] > 0$ if there is positive positive correlation between ξ and z in the baseline distribution μ^E. The stated cases follow. ■

Proof of Proposition A.5 and Proposition A.6. Consider a Pareto density $\mu^E(z)$ with minimum x and shape β. For brevity, we write x instead of z_{min} in this proof. Note that $\mathbf{I}(z) := \int_{\underline{z}}^{\infty} z^{\sigma-1}\mu^E(z)dz = x^\beta \frac{\beta}{\beta-(\sigma-1)} \underline{z}^{\sigma-1-\beta}$ with $\beta - (\sigma - 1) > 0$.

Step 1, calculate $E_{-1}, \underline{z}_{-1}$: The cutoff $\underline{z}_{-1}$ satisfies $\frac{f^e}{1} = \frac{\mathbf{I}(\underline{z}_{-1})}{\underline{z}_{-1}^{\sigma-1}} - \int_{\underline{z}_{-1}}^{\infty} \mu^E(z)dz$. From this follows

$$\frac{f^e}{1} = \frac{\mathbf{I}(\underline{z}_{-1})}{\underline{z}_{-1}^{\sigma-1}} - \int_{\underline{z}_{-1}}^{\infty} \mu^E(z)\, dz = \frac{\beta x^\beta}{\beta - (\sigma - 1)}\, \underline{z}_{-1}^{-\beta} - x^\beta \underline{z}_{-1}^{-\beta} = x^\beta \underline{z}_{-1}^{-\beta} \frac{\sigma - 1}{\beta - (\sigma - 1)}.$$

$$\Rightarrow \quad \underline{z}_{-1}^{\beta} = x^\beta \frac{\sigma - 1}{\beta - (\sigma - 1)} \cdot \frac{1}{f^e} \quad \Rightarrow \quad \underline{z}_{-1} = x \left\{ \frac{1}{f^e} \frac{\sigma - 1}{\beta - (\sigma - 1)} \right\}^{1/\beta}.$$

$$E_{-1} = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}_{-1}^{\sigma-1}}{\mathbf{I}(\underline{z}_{-1})} = \frac{\mathcal{I}}{\sigma} \left(\frac{\mathbf{I}(\underline{z}_{-1})}{\underline{z}_{-1}^{\sigma-1}} \right)^{-1} = \frac{\mathcal{I}}{\sigma} \left[\frac{\beta x^\beta}{\beta - (\sigma - 1)}\, \underline{z}_{-1}^{-\beta} \right]^{-1} = \frac{\mathcal{I}}{\sigma} \frac{\beta - (\sigma - 1)}{\beta}\, x^{-\beta} \underline{z}_{-1}^{\beta}.$$

$$\text{Using } x^{-\beta} \underline{z}_{-1}^{\beta} = \frac{\sigma - 1}{\beta - (\sigma - 1)} \cdot \frac{1}{f^e} \quad \Rightarrow \quad E_{-1} = \frac{\mathcal{I}}{\sigma} \frac{(\sigma - 1)}{\beta} \cdot \frac{1}{f^e}.$$

Step 2, truncate the firm measure to find the crisis measure (on impact): Immediately, $E_0 = 0$. From $\pi(\underline{z}_0, m_0) = 0 \Leftrightarrow \phi = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}_0^{\sigma-1}}{E_{-1}\mathbf{I}(\underline{z}_0)}$ we solve for $\underline{z}_0$:

$$\phi = \frac{\mathcal{I}}{\sigma} \frac{\underline{z}_0^{\sigma-1}}{E_{-1}\mathbf{I}(\underline{z}_0)} = \frac{\mathcal{I}}{\sigma} \frac{1}{E_{-1}} \left(\frac{\mathbf{I}(\underline{z}_0)}{\underline{z}_0^{\sigma-1}} \right)^{-1} = \frac{\mathcal{I}}{\sigma} \frac{1}{E_{-1}} \left[\frac{\beta x^\beta}{\beta - (\sigma - 1)}\, \underline{z}_0^{-\beta} \right]^{-1} = \frac{\underline{z}_0^{\beta}}{\underline{z}_{-1}^{\beta}}$$

$$\Rightarrow \quad \underline{z}_0^{\beta} = x^\beta \frac{\sigma - 1}{\beta - (\sigma - 1)} \cdot \frac{1}{f^e} \cdot \phi \quad \Rightarrow \quad \underline{z}_0 = x \left\{ \frac{\sigma - 1}{\beta - (\sigma - 1)} \cdot \frac{1}{f^e} \cdot \phi \right\}^{1/\beta}.$$

Step 3, calculate reentry: Now, in $t = 1, 2, ..., T^*$, fixed cost go down relative to the previous period. From the cutoff equation for entry, obtain directly

$$\underline{z}_t = x \left\{ \frac{f_t}{f^e} \frac{\sigma - 1}{\beta - (\sigma - 1)} \right\}^{1/\beta}$$

where $f_t = \phi^{1-t/T^*}$. Hence, the entry is given by

$$E_t = \frac{\mathcal{I}}{\sigma}\frac{1}{f_t^c}\frac{\underline{z}_t^{\sigma-1}}{\mathbf{I}(\underline{z}_t)} - \frac{E_{-1}\mathbf{I}(\underline{z}_0) + \sum_{\tau=1}^{t-1} E_\tau \mathbf{I}(\underline{z}_\tau)}{\mathbf{I}(\underline{z}_t)}.$$

Note that we can simplify the first term:

$$\frac{\mathcal{I}}{\sigma}\frac{1}{f_t^c}\frac{\underline{z}_t^{\sigma-1}}{\mathbf{I}(\underline{z}_t)} = \frac{\mathcal{I}}{\sigma}\frac{1}{f_t^c}\frac{\underline{z}_t^{\sigma-1}}{x^\beta \frac{\beta}{\beta-(\sigma-1)}\underline{z}_t^{\sigma-1-\beta}} = \frac{\mathcal{I}}{\sigma}\frac{1}{f_t^c}\frac{\beta-(\sigma-1)}{\beta x^\beta}\underline{z}_t^\beta.$$

Using $\underline{z}_t^\beta = x^\beta \left\{\frac{f_t^c}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}$

$$\Rightarrow \frac{\mathcal{I}}{\sigma}\frac{1}{f_t^c}\frac{\underline{z}_t^{\sigma-1}}{\mathbf{I}(\underline{z}_t)} = \frac{\mathcal{I}}{\sigma}\cdot\frac{1}{\beta}\cdot\frac{\sigma-1}{f^e} = \frac{\mathcal{I}}{\beta}\frac{\sigma-1}{\sigma}\frac{1}{f^e}.$$

Furthermore, the effect of pre-cycle incumbents is

$$E_{-1}\mathbf{I}(\underline{z}_0) = \frac{\mathcal{I}}{\sigma}\frac{\underline{z}_0^{\sigma-1}}{\phi} = \frac{\mathcal{I}}{\sigma}\frac{1}{\phi}\left[x^\beta\left\{\frac{\phi}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}\right]^{\frac{\sigma-1}{\beta}} = \frac{\mathcal{I}}{\sigma}\left\{x^\beta\frac{1}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{\frac{\sigma-1}{\beta}}\phi^{\frac{\sigma-1-\beta}{\beta}}.$$

In the summation above, we have $\mathbf{I}(\underline{z}_\tau)$. We solve for this object explicitly

$$\begin{aligned}
\mathbf{I}(\underline{z}_t) &= x^\beta\frac{\beta}{\beta-(\sigma-1)}\underline{z}_t^{\sigma-1-\beta} = x^\beta\frac{\beta}{\beta-(\sigma-1)}\left(\underline{z}_t^\beta\right)^{\frac{\sigma-1}{\beta}-1} \\
&= x^\beta\frac{\beta}{\beta-(\sigma-1)}\left[x^\beta\left\{\frac{f_t^c}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}\right]^{\frac{\sigma-1}{\beta}-1} \\
&= \frac{\beta}{\beta-(\sigma-1)}x^{\sigma-1}\left\{\frac{f_t^c}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{\frac{\sigma-1-\beta}{\beta}} \\
&= \frac{\beta}{\sigma-1}x^{\sigma-1}\left(\frac{\sigma-1}{\beta-(\sigma-1)}\right)^{\frac{\sigma-1}{\beta}}\left(\frac{f_t^c}{f^e}\right)^{\frac{\sigma-1-\beta}{\beta}} \\
&= \phi^{\frac{\sigma-1-\beta}{\beta}(1-\frac{t}{T^*})}\frac{\beta}{\sigma-1}f^e\left\{x^\beta\frac{1}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{\frac{\sigma-1}{\beta}}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{E_{-1}\mathbf{I}(\underline{z}_0)}{\mathbf{I}(\underline{z}_t)} &= \frac{\frac{\mathcal{I}}{\sigma}\left\{x^\beta\frac{1}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{\frac{\sigma-1}{\beta}}\phi^{\frac{\sigma-1-\beta}{\beta}}}{\phi^{\frac{\sigma-1-\beta}{\beta}(1-\frac{t}{T^*})}\frac{\beta}{\sigma-1}f^e\left\{x^\beta\frac{1}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right\}^{\frac{\sigma-1}{\beta}}} \\
&= \frac{\mathcal{I}}{\sigma}\frac{\sigma-1}{\beta}\frac{1}{f^e}\phi^{\frac{\sigma-1-\beta}{\beta}\frac{t}{T^*}}.
\end{aligned}$$

Also,

$$E_\tau\frac{\mathbf{I}(\underline{z}_\tau)}{\mathbf{I}(\underline{z}_t)} = E_\tau\phi^{\frac{\sigma-1-\beta}{\beta}\frac{t-\tau}{T^*}} \quad \text{for} \quad t = 1,\ldots,T^*.$$

Then, denoting $\pi := \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}$, we can write

$$E_t = E_{-1} - E_{-1}\pi^t - \sum_{\tau=1}^{t-1} E_\tau \pi^{t-\tau}$$

Step 4, solve the recursion and find m_∞: We can verify the following guess

$$E_t = E_{-1}(1-\pi)$$

From this, we can put together the measure m_τ:

$$\begin{aligned} m_\tau(z) &= \left(E_{-1}\mathbb{I}_{\{z \geq \underline{z}_0\}}(z) + \sum_{l=1}^{\tau} E_\tau \mathbb{I}_{\{z \geq \underline{z}_\tau\}}(z) \right) \mu^E(z) \\ &= E_{-1}\left(\mathbb{I}_{\{z \geq \underline{z}_0\}}(z) + \left(1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right) \sum_{l=1}^{\tau} \mathbb{I}_{\{z \geq \underline{z}_\tau\}}(z) \right) \mu^E(z) \end{aligned}$$

Substituting out the cutoffs $\underline{z}_0, \underline{z}_\tau$ and E_{-1} yields the expression for m^τ in the proposition.

Step 5, continuous-time convergence: Next, we consider what happens as the crisis approaches its continuous time limit. First we look at the tail of m_∞, i.e. at a value $z \geq \max \underline{z}_\tau = \underline{z}_0$ (recall that the cutoff monotone decreases after $t = 0$). Note the following limit: $N(1-u^{1/N}) \to -\log u$ for any $u > 0$ as $N \to \infty$. Applying this to our problem, we have

$$\begin{aligned} m^*_T(z) &= E_{-1}\left(\mathbb{I}_{\{z \geq \underline{z}_0\}}(z) + \left(1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right) \sum_{l=1}^{T^*} \mathbb{I}_{\{z \geq \underline{z}_\tau\}}(z) \right) \mu^E(z) \\ &= E_{-1}\left(1 + \left(1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right) T^* \right) \mu^E(z) \\ &\longrightarrow E_{-1} \cdot \mu^E(z) \cdot \left[1 - \frac{\sigma-1-\beta}{\beta}\log\phi\right] \quad \text{as } T^* \to \infty \end{aligned}$$

as claimed. Analyzing the left tail is harder. To this end, let $\alpha \in [0,1]$ and define

$$z_\alpha = x\left[\frac{\phi^{1-\alpha}}{f^e}\frac{\sigma-1}{\beta-(\sigma-1)}\right]^{1/\beta}.$$

Note that we can reach every $z \in [\underline{z}_{-1}, \underline{z}_0]$ in the left tail by varying α. We investigate the convergence of $m_{T^*}(z_\alpha)$. We have

$$\begin{aligned} m^*_T(z_\alpha) &= E_{-1}\left(\mathbb{I}_{\{z_\alpha \geq \underline{z}_0\}}(z_\alpha) + \left(1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right) \sum_{l=1}^{T^*} \mathbb{I}_{\{z_\alpha \geq \underline{z}_\tau\}}(z_\alpha) \right) \mu^E(z_\alpha) \\ &= E_{-1}\left(0 + \left(1 - \phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right) \sum_{l=\lceil \alpha T^* \rceil}^{T^*} \right) \mu^E(z_\alpha) \\ &\longrightarrow (1-\alpha) \cdot E_{-1} \cdot \mu^E(z_\alpha) \cdot \left[\frac{\beta-(\sigma-1)}{\beta}\log\phi\right] \quad \text{as} \quad T^* \to \infty. \end{aligned}$$

Finally get rid of α in the limit. Inverting z_α for α yields

$$\alpha = \left[1 - \frac{\log\left(\left(\frac{z_\alpha}{x}\right)^\beta f^e \frac{\beta-(\sigma-1)}{\sigma-1}\right)}{\log\phi}\right] = \left[1 - \beta\frac{\log\left(\frac{z_\alpha}{x}\right) - \log\left(\frac{\underline{z}_{-1}}{x}\right)}{\log\phi}\right]$$

Where we have used the formula for $\underline{z}_{-1}$. Substitute this expression back into the limit and the result follows.

Step 6, firm mass after the cycle M_{T^*}: Recall that the anti-cumulative of μ^E is denoted by p_E, and $p_E(a) = \left(\frac{x}{a}\right)^\beta$. Hence, we are looking for

$$\begin{aligned}
M_{T^*} &= E_{-1}p_E(\underline{z}_0) + \sum_{\tau=1}^{T^*} E_\tau p_E(\underline{z}_\tau) \\
&= \frac{\mathcal{I}}{\sigma}\frac{\beta-(\sigma-1)}{\beta}\cdot\phi^{-1} + \sum_\tau \frac{\mathcal{I}}{\sigma}\frac{\beta-(\sigma-1)}{\beta}\left[1-\phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right]\cdot\phi^{\frac{\tau}{T^*}-1} \\
&= \frac{\mathcal{I}}{\sigma}\frac{\beta-(\sigma-1)}{\beta}\cdot\phi^{-1}\left\{1+\left[1-\phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right]\cdot\sum_\tau\phi^{\frac{\tau}{T^*}}\right\} \\
&= \frac{\mathcal{I}}{\sigma}\frac{\beta-(\sigma-1)}{\beta}\cdot\phi^{-1}\left\{1+\left[1-\phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right]\cdot\phi^{\frac{1}{T^*}}\frac{1-\phi^{\frac{T^*}{T^*}}}{1-\phi^{\frac{1}{T^*}}}\right\}
\end{aligned}$$

Noting that

$$\frac{\left[1-\phi^{\frac{\sigma-1-\beta}{\beta}\frac{1}{T^*}}\right]}{1-\phi^{\frac{1}{T^*}}} \to \frac{\sigma-1-\beta}{\beta}$$

we get

$$M_{T^*} = \frac{\mathcal{I}}{\sigma}\frac{\beta-(\sigma-1)}{\beta}\cdot\phi^{-1}\left\{1+(\phi-1)\frac{\beta-(\sigma-1)}{\beta}\right\}$$

which equals the expression in the proposition.

■

Proof of Proposition A.7. Consider an economy in which both the distribution of entrants and of incumbents are kept general. Suppose, that the fixed cost have jumped up in the close past, hence the initial measure of incumbents, m_0 is truncated at some level z^*, but have since reverted back to some fixed level, f^c. This lower level of fixed cost is associated with cutoff $\underline{z} < z^*$. The mass of active firms is given by $M_t = \int m_t$, of which $M_t^I = \int m_{t-1}$ are incumbents. Like before, entrant masses are denoted by E_t. The distributional law of motion is given by

$$m_t(z) = m_{t-1}(z) + \sum_{\tau=1}^{t} E_t \mathbb{I}_{\{z\geq\underline{z}\}}(z)\mu^E(z), \tag{158}$$

where we can ignore the truncation of last period firms, since the cutoff does not move up over time. We impose a general equilibrium condition, labor markets must clear through

$$\mathcal{I}_t = \bar{L} + \Pi = \frac{\sigma}{\sigma-1}(\bar{L} - M_t f^c - E_t f^e). \tag{159}$$

Generally, let $\mathbb{E}_f(x) = \int x f(x)\ \mathrm{d}x / \int_0^\infty f(x)\ \mathrm{d}x$ be the expectation with respect to any non-negative function f, and let $p_f(a) = \int_a^\infty f(x)\ \mathrm{d}x / \int_0^\infty f(x)\ \mathrm{d}x$ be the anti-cumulative distribution function corresponding to f. E subscripts i.e., $\mathbb{E}_E$ and p_E, refer to the entry distribution, μ^E. The entry equation reads

$$E_t = \frac{1}{f^c}\frac{1}{\sigma-1}\left(\bar{L} - \sum_{1\leq\tau\leq t-1} p_E(\underline{z})E_\tau f^c - E_t(f^e + f^c p_E(\underline{z})) - M_0^I f^c\right)\underbrace{\frac{\underline{z}^{\sigma-1}}{\int_{\underline{z}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z}}_{\widetilde{Z}\equiv} \tag{160}$$

$$- \sum_{1\leq\tau\leq t-1} E_\tau - \underbrace{\frac{\int_{\underline{z}} z^{\sigma-1} m_0(z)\ \mathrm{d}z}{\int_{\underline{z}} z^{\sigma-1}\mu^E(z)\ \mathrm{d}z}}_{\widetilde{Z}_0\equiv} \tag{161}$$

$$= \frac{1}{f^c}\frac{1}{\sigma-1}\left(\bar{L} - \sum_{1\leq\tau\leq t-1} p_E(\underline{z})E_\tau f^c - E_t(f^e + f^c p_E(\underline{z})) - M_0^I f^c\right)\widetilde{Z} - \sum_{1\leq\tau\leq t-1} E_\tau - \widetilde{Z}_0 \tag{162}$$

$$\Longleftrightarrow$$

$$E_t = \underbrace{\frac{\frac{\widetilde{Z}}{\sigma-1}\frac{\bar{L}-M_0 f^c}{f^c} - \widetilde{Z}_0}{1 + \frac{\widetilde{Z}}{\sigma-1}(f^e/f^c + p_E(\underline{z}))}}_{A\equiv} - \left[\sum_{1\leq\tau\leq t-1} E_\tau\right]\underbrace{\left[\frac{1 + p_E(\underline{z})\frac{\widetilde{Z}}{\sigma-1}}{1 + \frac{\widetilde{Z}}{\sigma-1}(f^e/f^c + p_E(\underline{z}))}\right]}_{b\equiv} \tag{163}$$

from which one deduces that

$$E_t = (1-b)^{t-1}A. \tag{164}$$

We can rewrite the coefficients as (letting p_0 be the anti-cumulative of m_0)

$$A = \frac{(\bar{L}/f^c - M_0) - \mathbb{E}_{m_0}\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1}|z\geq\underline{z}\right](\sigma-1)\,p_0(\underline{z})}{f^e/f^c + \left(\mathbb{E}_E\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1}|z\geq\underline{z}\right](\sigma-1)+1\right)p_E(\underline{z})} \tag{165}$$

$$1-b = \frac{f^e/f^c}{f^e/f^c + \left(\mathbb{E}_E\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1}|z\geq\underline{z}\right](\sigma-1)+1\right)p_E(\underline{z})}. \tag{166}$$

Therefore, finally

$$E_t = \frac{f^c}{f^e}(1-b)^t \cdot \left[(\bar{L}/f^c - M_0) - \mathbb{E}_{m_0}\left[\left(\frac{z}{\underline{z}}\right)^{\sigma-1}|z\geq\underline{z}\right](\sigma-1)\,p_0(\underline{z})\right]. \tag{167}$$

■

Proof of Proposition A.8. In a state of entry, we obtain

$$\frac{f^e}{f^c} = \int_{\underline{z}}^{\infty} \left[\frac{\mathbb{E}_\psi\big[(z+\varepsilon)^{\sigma-1}\big]}{\mathbb{E}_\psi\big[(\underline{z}+\varepsilon)^{\sigma-1}\big]} - 1 \right] \mu^E(z)\, dz = \frac{\widetilde{Z}(\mu,\psi)}{\mathbb{E}_\psi\big[(\underline{z}+\varepsilon)^{\sigma-1}\big]} - p_E(\underline{z}), \tag{168}$$

from which we see immediately that cutoffs are independent of incumbents. Like in the baseline, equilibrium is established through entry E_ε which shifts $\widetilde{Z}$. For the second part of the proposition, note that a firm which enters with productivity z in period k of a cycle phase $\tau \in \{1,2,3\}$ is still alive in period $t > k$ with probability $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon)^{t-k}$, which converges to 0 if and only if $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon) > 0$. Therefore, all incumbents at the beginning of the period for whom $\mathbb{P}_\psi(z+\varepsilon < \underline{z}_\varepsilon) > 0$ holds true will also exit eventually, and only if there is a positive measure of incumbents who never exit, path dependency holds. ■

Proof of Proposition A.9. For (ii), a very similar argument to Proposition 2 applies. Note that individual firms' profits in eq. (71) are independent of f^p. To see this, note that the only element in eq. (71) that could depend on f^p is L^p. Using eq. (68) in steady state, i.e. with $E = 0$, we can write L^p as:

$$L^p = \bar{L} - M f^c - N f^p \tag{169}$$

Integrating eq. (66), and substituting it in the above, we obtain:

$$L^p = \bar{L} - M f^c - \frac{L^p}{(\eta-1) f^p} f^p. \tag{170}$$

which gives us that:

$$L^p = \frac{\eta - 1}{\eta}(\bar{L} - M f^c) \tag{171}$$

As firm profits are independent of f^p, entry and exit decisions are unaffected by f^p. Therefore, movements in f^p do not change $(m, \underline{z})$, and there is no path-dependency and, thus, no long-run effects.

For (i) (a), firstly, note that in PE $\mathcal{I}$ is exogenously fixed. Therefore, $\mathcal{I}_3 = \mathcal{I}_1$, and given that $L^p_\tau = \frac{\sigma-1}{\sigma}\mathcal{I}_\tau$, also $L^p_3 = L^p_1$. Secondly, note that the proof of Proposition 1 applies with $z^{\phi-1}$ instead of $z^{\sigma-1}$, which has no material effect. Therefore, following the steps of Proposition 1, we obtain that $\mathcal{Z}_3 = \mathcal{Z}_1$ and $M_3 < M_1$. In addition, given that by integrating eq. (66) we have $N = \frac{L^p}{(\eta-1)f^p}$, we obtain that $N_3 = N_1$. The condition on the q's immediately follows.

For (i) (b), note that the proof of Proposition 4 applies with $z^{\phi-1}$ instead of $z^{\sigma-1}$, which has no material effect, and with:

$$R_3 - R_1 = \frac{\sigma}{\sigma - 1}\frac{\eta - 1}{\eta} f^c (M_1 - M_3) \tag{172}$$

where $R_3 - R_1$ is obtained by using that $R_\tau = \frac{\sigma}{\sigma-1} L^p_\tau$. Using the argumentation in the proof of

Proposition 4, we obtain that $M_3 < M_1$ and $\mathcal{Z}_3 > \mathcal{Z}_1$. For the multi-product setting, we have that:

$$L^p = \frac{\eta - 1}{\eta}(\bar{L} - Mf^c) \tag{173}$$

$$N = \frac{\bar{L} - Mf^c}{\eta f^p} \tag{174}$$

where to obtain N, we substitute L^p in $N = \frac{L^p}{(\eta-1)f^p}$. Therefore, we can conclude that: $L_3^p > L_1^p$, $N_3 > N_1$. The condition on the q's immediately follows.

For (i) (c), firstly note that: $Y_\tau^{CES} := N_\tau^{q_V} L_\tau^p \mathcal{Z}_\tau$ and thus:

$$Y_\tau^{CES} \propto (\bar{L} - Mf^c)^{1+q_V}\mathcal{Z}_\tau.$$

Therefore, the only changes to eq. (20) are that within the parenthesis:

1. instead of $(q - q^{CES})$ we have $(q_F - \frac{1}{\sigma-1}) - (q_V - \frac{1}{\eta-1})$
2. for $\frac{\partial \log Y_3^{CES}}{\partial \log M_3}$ we just have that the labor cleansing effect is amplified. Recall that in the single-product firm case, we had $Y_\tau^{CES} = (\bar{L} - Mf^c)\mathcal{Z}_\tau$. However, the elasticity is still increasing (in absolute terms) in M_3.

Therefore, the proof of Proposition 5 applies, with the only difference that the q-interval in which the long-run effects are dependent on f_h^c needs to have that $q_\circ - \frac{1}{\sigma-1} > q_V - \frac{1}{\eta-1}$, as stated in the proposition.

■

Proof of Proposition A.10. We define the following proof-specific objects:

$$\widetilde{Z}_\tau := \int z^{\sigma-1} m_\tau(z)dz, \quad a := \alpha(q - q^{\text{CES}}), \quad b := \frac{\alpha + 1 - \sigma}{\sigma - 1}, \quad \widetilde{\sigma} := \sigma^\alpha(\sigma-1)^{1-\alpha}.$$

Let the technology for entry and per-period fixed operating costs be Cobb–Douglas in the final good and labor:

$$B(Y, L) \ = \ \kappa(\alpha)\, Y^\alpha L^{1-\alpha}, \qquad \kappa(\alpha) = \alpha^\alpha(1-\alpha)^{1-\alpha}, \quad \alpha \in [0,1].$$

Let (P_t, w_t) denote the prices of the final good and labor, then the cost of the bundle B is

$$\Phi_t(P_t, w_t) \ = \ P_t^\alpha w_t^{1-\alpha}.$$

The firm requires f^e units of B to enter and f^c units of B, per period, to produce. As in Proposition 4, we analyze steady-state output, therefore, there is no payment of entry costs. With labor as the numeraire ($w_t = 1$), the nominal expenditure on fixed cost payments at time τ is

$$f^c \cdot \Phi_\tau(P_\tau, 1) \ = \ f^c\, P_\tau^\alpha.$$

This nests two special cases. If $\alpha = 0$, then the fixed costs are paid only in units of labor, and the nominal expenditure is $w_\tau f^c = f^c$. If $\alpha = 1$, then the fixed costs are paid only in final-good units, and the nominal expenditure is $f^c P_\tau$.

Change in output after the crisis The zero-profit condition at time τ can be written as

$$\begin{aligned} \frac{L^p_\tau \, \underline{z}_\tau^{\sigma-1}}{(\sigma-1)\, P_t^\alpha \, f^c_\tau} &= \mathcal{Z}_\tau^{\sigma-1} \\ \frac{L^p_\tau}{\sigma^\alpha(\sigma-1)^{1-\alpha} f^c_\tau} M_\tau^{\alpha(q-\frac{1}{\sigma-1})} \underline{z}_\tau^{\sigma-1} &= \mathcal{Z}_\tau^{\sigma-\alpha-1} \end{aligned} \tag{175}$$

Dividing the zero-profit conditions at $\tau = 3$ and $\tau = 1$, and using that $f^c_3 = f^c_1$, we obtain:

$$\frac{L^p_3}{L^p_1} \left(\frac{M_3}{M_1}\right)^a \left(\frac{\underline{z}_3}{\underline{z}_1}\right)^{\sigma-1} = \left(\frac{\mathcal{Z}_3}{\mathcal{Z}_1}\right)^{\sigma-\alpha-1}$$

Using Lemma 1, which also applies in this case, and implies $\underline{z}_3 = \underline{z}_1$, we obtain:

$$\begin{aligned} \frac{L^p_3}{L^p_1} \left(\frac{M_3}{M_1}\right)^a \frac{\mathcal{Z}_3}{\mathcal{Z}_1} &= \left(\frac{\mathcal{Z}_3}{\mathcal{Z}_1}\right)^{\sigma-\alpha} \\ \frac{Y_3}{Y_1} \left(\frac{M_3}{M_1}\right)^{(\alpha-1)(q-q^{CES})} &= \left(\frac{\mathcal{Z}_3}{\mathcal{Z}_1}\right)^{\sigma-\alpha} \end{aligned}$$

Taking logs, we obtain

$$\Delta \log Y = (\sigma - \alpha)\Delta \log \mathcal{Z} + (1-\alpha)(q - q^{CES})\Delta \log M. \tag{176}$$

Note that we can also rewrite eq. (176) as:

$$\Delta \log Y = \frac{\sigma - \alpha}{\sigma - 1} \Delta \log \widetilde{Z} + (1-\alpha)(q - q^{CES})\Delta \log M, \tag{177}$$

as per the proposition.

$\mathbf{q}^\star$ When fixed costs are entirely paid in units of output, i.e. $\alpha = 1$, we have that

$$\Delta \log Y = (\sigma - 1)\, \Delta \log \mathcal{Z} = \Delta \log \widetilde{Z},$$

thus $\Delta \log Y = 0 \iff \Delta \log \widetilde{Z} = 0$, which holds at $q = q^{\text{CES}}$. Hence $q^\star(1) = q^{\text{CES}}$. To see that $\Delta \log \mathcal{Z} = 0$ at $q = q^{CES}$ when $\alpha = 1$, start from eq. (175), and substitute $q = q^{CES} = \frac{1}{\sigma-1}$ and $\alpha = 1$, then

$$\frac{L^p_\tau}{\sigma f^c_\tau} \underline{z}_\tau^{\sigma-1} = \mathcal{Z}_\tau^{\sigma-2}$$

Lemma 1, which still applies here, together with $f_3 = f_1$ and $L^p_\tau = \bar{L} \; \forall \tau$, as fixed costs are not paid in labor with $\alpha = 1$, immediately give the result. Starting from eq. (176), for $\alpha \in [0, 1)$, the $q^\star(\alpha)$ solving $\Delta \log Y = 0$ satisfies

$$q^\star(\alpha) \;=\; q^{\text{CES}} \;-\; \frac{\sigma-\alpha}{1-\alpha}\,\frac{\Delta \log \mathcal{Z}\big(q^\star(\alpha),\alpha\big)}{\Delta \log M\big(q^\star(\alpha),\alpha\big)}. \tag{178}$$

Equation (178) implies that $q^\star(\alpha) > q^{\text{CES}}$ when $\alpha < 1$. The statement follows from $\Delta \log M < 0$ and $\Delta \log \mathcal{Z} > 0$ after the crisis, because of the labor-saving effect as soon as $\alpha < 1$.

Therefore, as per the proposition, we have that for each $\alpha \in [0,1]$, there exists a $q^\star(\alpha)$ such that $\Delta \log Y = 0$. Moreover, $q^\star(\alpha) \geq q^{\text{CES}}$ for all α, with $q^\star(1) = q^{\text{CES}}$.

$\mathbf{q} \neq \mathbf{q}^\star$ When $\alpha = 0$, we characterize the change in output after the crisis globally, see Proposition 4. For $\alpha \in (0,1]$, we can characterize the change in output locally around $q^\star(\alpha)$, to first order. The zero-profit condition at $t=\tau$, i.e. eq. (175), can be expressed as:

$$\frac{L^p_\tau\, \underline{z}_\tau^{\sigma-1}}{\widetilde{\sigma}} \widetilde{Z}^b_\tau M^a_\tau = f^c_\tau.$$

By taking logs, we obtain:

$$\log L^p_\tau + (\sigma-1)\log \underline{z}_\tau - \log \widetilde{\sigma} + b \log \widetilde{Z}_\tau + a \log M_\tau = \log f^c_\tau. \tag{179}$$

By subtracting eq. (179) at $t=1$ to $t=3$, we obtain:

$$\Delta \log L^p + b\Delta \log \widetilde{Z} + a\Delta \log M = 0, \tag{180}$$

where $\Delta \log X := \log X_3 - \log X_1$, and we used that $f^c_3 = f^c_1$ and Lemma 1. We can solve for $\Delta \log \widetilde{Z}$ in eq. (180), which gives us:

$$\Delta \log \widetilde{Z} = -\frac{1}{b}\left[a(q)\Delta \log M + \Delta \log L^p\right],$$

where we explicitly note that a is a function of q. We can now substitute for $\Delta \log \widetilde{Z}$ in eq. (177), obtaining:

$$\Delta \log Y = \left[(1-\alpha)(q - q^{CES}) - \frac{\sigma-\alpha}{\sigma-1}\frac{a(q)}{b}\right]\Delta \log M - \frac{\sigma-\alpha}{\sigma-1}\frac{1}{b}\Delta \log L^p. \tag{181}$$

Then, the first-order effect of q on $\Delta \log Y$ locally around $q^\star(\alpha)$ is:

$$\frac{\partial}{\partial q}\Delta \log Y(q,\alpha)\bigg|_{q=q^\star(\alpha)} = \left[(1-\alpha) - \frac{(\sigma-\alpha)}{(\sigma-1)}\frac{\alpha}{b}\right]\Delta \log M\big|_{q^\star(\alpha)}. \tag{182}$$

Therefore, given that $\Delta \log M < 0$, we have that if $\sigma > \alpha+1$, and thus $b = \frac{\alpha+1-\sigma}{\sigma-1} < 0$, then

$$\frac{\partial}{\partial q}\Delta \log Y(q,\alpha)\bigg|_{q=q^\star(\alpha)} < 0, \tag{183}$$

implying that

$$q \gtrless q^{\star}(\alpha) \Rightarrow \Delta \log Y(q) \lessgtr 0 \quad \text{for } q \text{ in a neighborhood of } q^{\star}(\alpha) \text{, to first order.}$$

■

C Data and Empirical Analysis

C.1 WIOD Data

The WIOD data contains information on gross output at current prices for all industries in a given year. It also provides a version of the same information at previous year prices (PYP). Define $S_t = P_t Y_t$ and $S_t^{t-1} = P_{t-1} Y_t$. We can write

$$\log \frac{S_t^{t-1}}{S_{t-1}^{t-1}} = \log P_{t-1} + \log Y_t - \log P_{t-1} - \log Y_{t-1} = \\ = \log Y_t - \log Y_{t-1} = \Delta \log Y_t. \tag{184}$$

C.2 Robustness analysis

Weighting by industry We now estimate q using weights based on the industry median log total sales across countries and years. This approach gives greater influence to industries with higher median total sales. As it can be seen from Table A.1, the point estimates are very similar to section 5, and q is significantly higher than q^{CES}.

Table A.1: Estimation Results with Industry-Size Weights

	(D)		(E)		(F)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^i_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log L^{p,i}_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log Y^i_{c,t}$
$\eta^{+,i}_{c,t}$	0.0890***		0.101***		
	(0.0209)		(0.0192)		
$\eta^{+,i}_{c,t-1}$	0.0967***		0.0646***		
	(0.0225)		(0.0191)		
$\Delta \log \mathcal{I}^i_{c,t}$		1.460***			1.535***
		(0.174)			(0.141)
$\Delta \log L^{p,i}_{c,t}$				1.648***	1.535***
				(0.236)	(0.141)
$P(q \leq q^{CES})$		.074		.031	.010
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	10067	10067	10083	10083	10067
F-Stat	25	70	25	49	
R^2	0.0323	0.0864	0.0426	0.0881	
F-Stat eq.(1)					1300
F-Stat eq.(2)					127
R^2 eq.(1)					0.0866
R^2 eq.(2)					0.0875

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

Single instrument In our main analysis, we estimate q using both lagged and contemporary positive demand shocks as instruments for income and labor. As shown in Tables A.2 and A.3, our results are robust to the choice of instruments.

Table A.2: Estimation Results with only lagged positive demand shocks

	(G)		(H)		(I)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^{i}_{c,t}$	$\Delta \log Y^{i}_{c,t}$	$\Delta \log L^{p,i}_{c,t}$	$\Delta \log Y^{i}_{c,t}$	$\Delta \log Y^{i}_{c,t}$
$\eta^{+,i}_{c,t-1}$	0.104***		0.112***		
	(0.0188)		(0.0154)		
$\Delta \log \mathcal{I}^{i}_{c,t}$		1.680***			1.622***
		(0.255)			(0.165)
$\Delta \log L^{p,i}_{c,t}$				1.568***	1.622***
				(0.216)	(0.165)
$P(q \leq q^{CES})$		.032		.047	.006
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	15507	15507	15526	15526	15505
F-Stat	31	44	53	53	
R^2	0.0254	0.1235	0.0353	0.1237	
F-Stat eq.(1)					7
F-Stat eq.(2)					10
R^2 eq.(1)					0.12353
R^2 eq.(2)					0.12354

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

Table A.3: Estimation Results with only contemporary positive demand shocks

	(J)		(K)		(L)
	(1)	(2)	(3)	(4)	(5)
	$\Delta \log \mathcal{I}^i_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log L^{p,i}_{c,t}$	$\Delta \log Y^i_{c,t}$	$\Delta \log Y^i_{c,t}$
$\eta^{+,i}_{c,t}$	0.114***		0.104***		
	(0.0179)		(0.0169)		
$\Delta \log \mathcal{I}^i_{c,t}$		1.745***			1.816***
		(0.235)			(0.186)
$\Delta \log L^{p,i}_{c,t}$				1.919***	1.816***
				(0.316)	(0.186)
$P(q \leq q^{CES})$		.011		.012	.001
Year FE	Yes	Yes	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes	Yes	Yes
N	15029	15029	15048	15048	15028
F-Stat	41	55	38	37	
R^2	0.0264	0.0689	0.0317	0.0693	
F-Stat eq.(1)					6
F-Stat eq.(2)					5
R^2 eq.(1)					0.0690
R^2 eq.(2)					0.0689

Robust Standard Errors (HC1).

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: $P(q \leq q^{CES})$ is computed under the null hypothesis $H_0 : q \leq q^{CES}$. When we estimate the system jointly using 3SLS, $P(q \leq q^{CES})$ is computed using the estimated coefficient and standard error of the income regressor. The F-statistic for the jointly estimated model is approximated by the model's Wald test statistic, which follows a χ^2 distribution, divided by the model degrees of freedom. We impose a common coefficient in the joint estimation, as we fail to reject, at the 10% significance level, the hypothesis that the coefficients on labor and income are the same.

D Additional Results Quantitative Model

In addition, with Table A.4 and Figures A.3, A.4, and A.5, we provide an analogue of Table 4 and Figures 6, 7, and 8 for $q = 0.3$, which is the love of variety in production estimated by Baqaee et al. (2023). As in the main figures, random exit shocks ensure a unique long-run steady state. Any residual in the panels reflects the 25-year plotting window.

Table A.4: Steady-state welfare gains and recession welfare costs (CEV, %)

A. Steady State	*CEV (%)*
ss policy	**+3.40%**

B. Recession	*CEV (%)*
Welfare cost of recession	**-1.56%**
Variety	-0.97%
Production labor	-0.39%
TFP	-0.21%
Welfare cost of recession with ss policy	**-1.53%**
Variety	-0.94%
Production labor	-0.40%
TFP	-0.19%
Welfare cost of recession with ss and cycle policy	**-1.50%**
Variety	-0.44%
Production labor	-0.85%
TFP	-0.21%

Notes: the top subsection reports the *steady-state* welfare gain from adopting the steady-state planner relative to a laissez-faire economy. All other rows report the *welfare cost of the recession* relative to the corresponding steady state. The decomposition of the total welfare cost shows the contribution of the unique components of output, i.e. Variety Mass, Production labor, and TFP, to the total CEV. The results are reported using the q from Baqaee et al. (2023). Numbers are shown as CEV percentages.

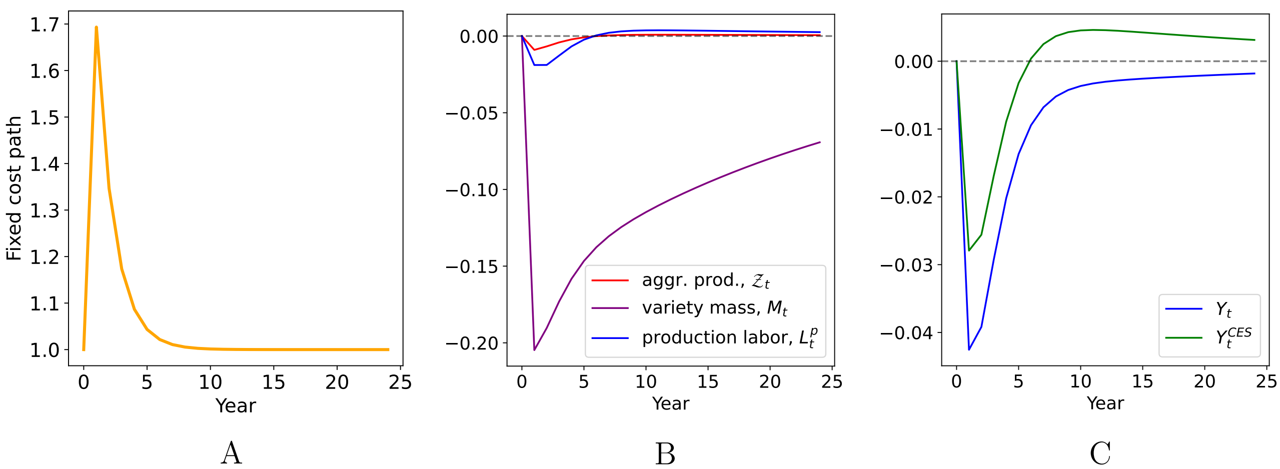

Figure A.3: Fixed-cost shock path and impulse responses in a laissez-faire economy. Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.

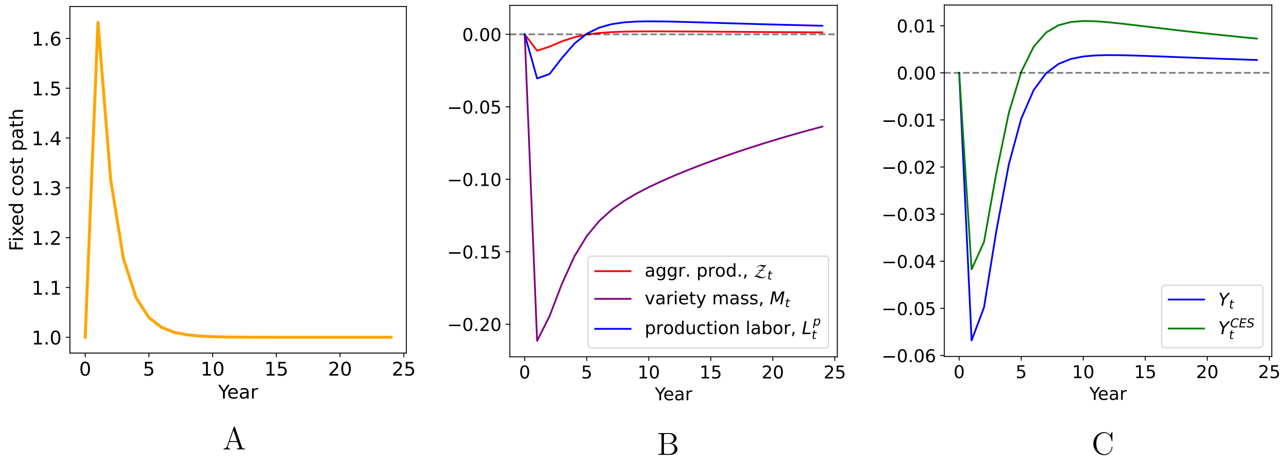

Figure A.4: Fixed-cost shock path and impulse responses with a steady-state planner. Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.

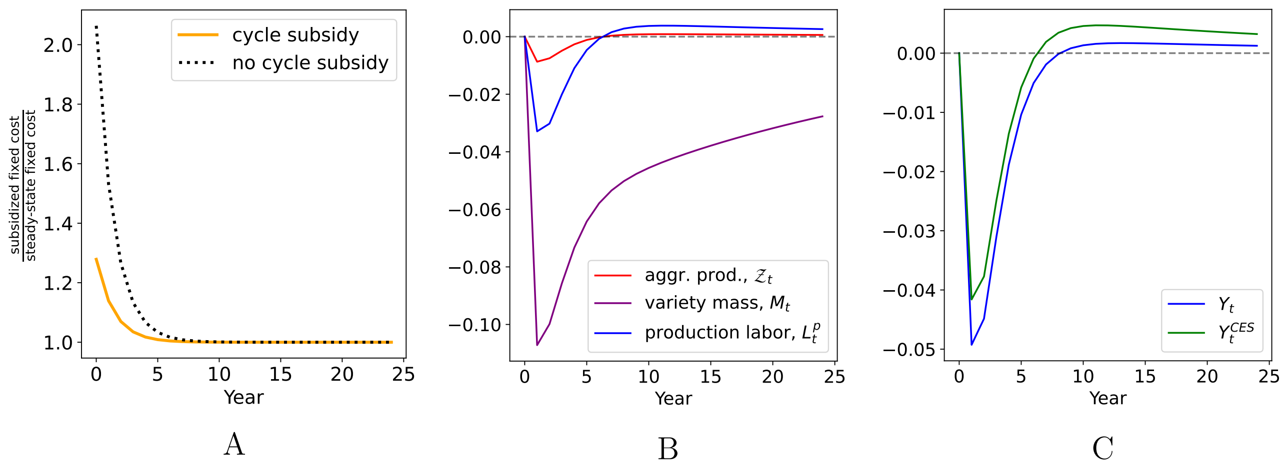

Figure A.5: Fixed-cost shock path and impulse responses with a cycle planner (ss + cycle). Panel A displays the fixed-cost shock and its reversion calibrated as per 6.1. Panel C and B report the impulse responses–log deviations from the pre-shock steady state–of output and its components: variety mass M_t, production labor L_t^p, and TFP $\mathcal{Z}_t$, respectively.