# Unequal Societies with Equality of Opportunity Internet and the Rise of Political Tribalism in Europe

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# Unequal Societies with Equality of Opportunity

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**Abstract**

Is long-term income inequality consistent with equality of opportunity (EOp)? We study a dynamic economy in which one generation's inequality shapes the socio-economic status, and hence the circumstances, of the next, with status a positional good governing the returns to effort. We show that a maximin EOp policy cannot sustain the welfare of full equality: from every starting point the intergenerational welfare floor lies strictly below its egalitarian level. A myopic, within-generation policy drives an initially more equal economy to an ever more unequal steady state. If the date of birth is taken as a circumstance too, the intergenerational EOp policy remains consistent with high and *persistent* inequalities in welfare, income, and status. EOp policies produce results that seem at odds with the general EOp ethic. For they may lead inequalities to remain constant or even increase over time, even though these inequalities are not the product of individual responsible choices.

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# 1 Introduction

Equality of opportunity (EOp) is the most widely accepted form of egalitarianism: unequal outcomes are held to be acceptable as long as they reflect people's choices rather than their circumstances, the factors beyond individual control. Yet not all circumstances are fixed.

As Heckman (2013, p. 10) notes, in many societies "birth is becoming fate": the family one is born into increasingly shapes the opportunities one faces. When the circumstances of one generation depend on the outcomes of the last, does a policy that equalises opportunity actually reduce inequality? We show that it need not: a maximin EOp policy does not in general reduce inequality, and in a range of cases increases it, even though the resulting inequalities are not the product of responsible choices. The question is not merely theoretical. Inequality of opportunity is estimated for many countries (see https://geom.ecineq.org), and a large empirical literature relates it to income inequality and to other outcomes (Van de gaer et al., 2001; Roemer et al., 2003; Checchi and Peragine, 2005; Bourguignon et al., 2007).

The central idea of the modern theory of equality of opportunity is that the economic outcomes of individuals are determined, broadly speaking, by three sets of variables: agents' circumstances (a set of factors beyond individual control, such as gender, race, family of origin, genetic make up), their choices, and government policies.[1] The *EOp ethic* requires the elimination of all inequalities arising from factors beyond agents' control. However, unlike outcome egalitarianism, EOp theory considers inequalities due to agents' choices morally justified. This ethic must be distinguished from an EOp *policy*, the redistributive scheme a polity adopts under the constraints of what is politically and informationally feasible. The distinction is immaterial in a static setting but, as we show, the two come apart once inequality is transmitted across generations.

The EOp ethic can be articulated in many ways, and the literature is better described as a broad family of approaches sharing a common intuition while differing on the notion of opportunity, the definition of circumstances, the ex ante versus ex post perspective, and so on.[2] This inherent multiplicity in the specification of the EOp ethic is particularly evident in one of the most prominent approaches in the literature, which has been developed by John Roemer in a series of seminal contributions (Roemer, 1996, 1998; Roemer et al., 2003). The starting point of Roemer's theory is precisely the acknowledgement of the existence of a variety of views concerning individual responsibility and therefore what constitutes equal opportunity. "My purpose is to propose an algorithm which will enable a society …to translate any such view about personal accountability into a social policy that will implement a kind or degree of equal opportunity consonant with that view" (Roemer, 1998, p.2).

[1]The modern theory of EOp originates from Rawls (1971); landmark contributions include Dworkin (1981), Cohen (1989), and Arneson (1989), with surveys in Roemer (1996) and Roemer and Trannoy (2015).

[2]The literature is vast. A representative but far from exhaustive sample includes Van de gaer (1993), Fleurbaey (1995, 2008), Bossert (1995), and Fleurbaey and Maniquet (2011). For a detailed survey, see Roemer and Trannoy (2016) and Ramos and Van de gaer (2016).

According to this algorithm, first, the set of circumstances, $\Theta_c$, and the set of responsible factors, $\Theta_r$, should be identified. The set $\Theta_c$ partitions society into *types*: all agents sharing the same circumstances, such as gender, race, or genetic make up, belong to a given type. Then, within a set of feasible policies $\Phi$, society should choose the policy $\phi^{EOp} \in \Phi$ which equalises the distribution of the relevant outcome (welfare, income, education, and so on) across different types. If $\Theta_r$ corresponds to effort, then Roemer proposes that agents belonging to different types but exerting the same degree of effort, that is, agents at the same centile of the effort distribution of their own types, obtain the same outcome. Because it does not fix the sets $\Theta_c, \Theta_r$, Roemer's approach spans a wide range of views: the smaller the set of factors deemed beyond individual control, the less egalitarian the EOp policy $\phi^{EOp}$, with the laissez faire policy as the limiting case in which circumstances play no role.

By avoiding a purely normative, a priori specification of the different elements of the theory, Roemer's approach is therefore fundamentally *political*, in two respects. First, it provides a clear, rational ground for political debate: the EOp policy $\phi^{EOp}$ implements the EOp ethic once a society collectively defines the sets $\Theta_c, \Theta_r$ through a political process. Second, it takes into account political constraints in the definition of the set of feasible policies $\Phi$ from which $\phi^{EOp}$ is chosen. For example, progressive taxation, or more generally type-dependent policies may be politically infeasible in a given society (Roemer et al., 2003). Given this indeterminacy, the empirical and policy literature has often adopted a minimalist approach, identifying $\phi^{EOp}$ from a limited set of policies and a small set of circumstances. This fits the political dimension of EOp, since consensus is easier on fewer types and less divisive policies. Even so, these applications have typically shown that significant redistribution is necessary: the resulting $\phi^{EOp}$ may not fully realise the EOp ethic, but it can be read as a lower bound on optimal redistribution that egalitarians may accept as a political compromise.

In this paper, we raise some doubts on the latter conclusion, and on certain features of the EOp approach more generally. In particular, we show that the idea that the minimalist approach may provide at least a first, partial step in the direction of implementing the EOp ethic works at most in a *static* framework. Once dynamic effects are taken into account, the simplifying assumptions adopted in much of the literature may yield counterintuitive and undesirable implications *from an EOp perspective*.

We consider an infinite-horizon, dynamic economy with nonoverlapping generations. In every period, agents produce one good and their productivity depends on the effort they exercise and on their status, the socioeconomic class they are born into, which is a positional good and partitions agents into two groups, those born into rich families and those born into poor ones. Our setting is close to Piketty (1995, 1998), Roemer and Unveren (2016), and Roemer and Veneziani (2004): unlike the first three we rule out intergenerational mobility but make class incomes endogenous, as in Roemer and Veneziani (2004), and unlike Roemer and Veneziani (2004) we give effort an explicit role and treat status as a positional good.

Agents have the same skills and the same preferences over consumption and effort, and choose effort in order to maximise utility.[3] In this abstract model, in principle the egalitarian polity can take into account two circumstances which characterise agents: their date of birth and their socioeconomic status (henceforth, SES).

What is the EOp programme in this context? What is the objective function, and what is the feasible set of policies $\Phi$? In Roemer's political approach, there is no a priori, unambiguously correct answer to either of these questions. As Roemer notes, the objective function will emerge after the polity has decided the set of circumstances $\Theta_c$. Further, because it is empirically impossible to explore the entire set of feasible policies $\Phi$, political debate has to define also the set of policies to choose from. We analyse two EOp programmes: in the *Myopic EOp programme* (MEOp), the egalitarian polity considers SES as the only circumstance and chooses a linear taxation and redistribution scheme to maximise the minimum level of welfare of the agents belonging to the same generation; in the *Intergenerational EOp programme* the planner equalises opportunities across generations too. Welfare is the natural equalisandum here, though we also track income and status.[4] Although it may seem morally arbitrary to ignore date of birth as a relevant circumstance, the political nature of Roemer's EOp approach provides a natural justification for considering MEOp: given that future generations cannot participate in the political process, current agents may well decide to ignore date of birth as a relevant circumstance and equalise opportunities *within each generation*.[5]

We focus on linear taxation for three reasons. First, linear tax-and-redistribute schemes are standard in the EOp literature following Roemer's formalisation (Roemer, 1996, 1998; Roemer et al., 2003; Roemer and Veneziani, 2004; Roemer and Unveren, 2016; Roemer and Trannoy, 2016) and in similar approaches (Piketty, 1995, 1998; Bénabou and Tirole, 2006). Second, as Roemer argues, type-dependent policies, such as, most obviously, progressive taxation, may be politically difficult to implement. Third, the restriction to a flat tax is conservative: although statutory schedules are progressive, *effective* rates turn regressive at the very top, where the avoidance opportunities open to the very rich leave billionaires paying lower rates than the upper-middle class (Saez and Zucman, 2019; Zucman, 2024). Since our results concern precisely such small elites, our welfare-maximising flat rate is more redistributive than the schedules the rich actually face, so the inegalitarian dynamics we identify arise under an assumption that, if anything, favours equality.

The central finding concerns what the EOp policy does *not* achieve: from no starting point does it sustain the welfare of full equality, and the optimal dynamics we characterise

[3] We rule out skill and preference heterogeneity, which is secondary for our argument and avoids some definitional complications in EOp theory (Fleurbaey, 1995; Bossert, 1995).

[4] Ruling out within-type heterogeneity avoids the complications of equalising the *distributions* of achievements across types (see Roemer (1998), Fleurbaey (2008), and Van de gaer (1993)).

[5] During 1997-2010, the UK Labour Party in government pursued policies aimed at equalising opportunities, including reducing child poverty, but focused on currently living agents, with little attention to intergenerational effects (see Department of Social Security (1999) and Hills and Stewart (2005)).

leave the economy short of equal status. At the solution of the MEOp, $\phi^{EOp}$ may well imply a *monotonic increase* in (welfare, income and status) inequalities within each generation over time. Poor agents become progressively poorer and if the initial proportion of poor agents in the population is relatively high, the economy tends to a steady state with extremely high inequalities. If the polity decides to take into account date of birth as a circumstance too, then the welfare of the poor need no longer decline over time. Nonetheless, the intergenerational EOp policy remains consistent with high and *persistent* inequalities in welfare, income, and status: from every starting point its welfare floor lies strictly below the level of full equality, and neither of the policies we characterise takes the economy to equal status. This polarisation channel links our analysis to the wider debate on the concentration of income and wealth at the top, the '1% versus 99%' that has become a focal point of discussion (Atkinson et al., 2011; Stiglitz, 2012; Piketty, 2014; Milanovic, 2016).

We believe that these results are problematic from the viewpoint of EOp theory. The issue is not that, as is well known, equality of *opportunity* may be consistent with (potentially large) *outcome* inequalities. Nor is it that EOp policies may be even less egalitarian than unfettered market outcomes (see, for example, Roemer's (1998) discussion of unemployment insurance). Rather our results suggest that EOp policies may produce results that seem at odds with the general EOp ethic. For, if one takes the political approach to EOp seriously, and equalises opportunity subject to the relevant political constraints on feasible taxation and given political decisions on individual circumstances, then this may lead inequalities to remain constant or even increase over time, *even though these inequalities are not the product of individual responsible choices*.

Both policies maximise the welfare of the worst off across classes *while taking SES as given*. Yet, except in the first period, SES is not a fact of nature, unlike date of birth: it is endogenous to the problem, and it is unclear that an EOp ethic should take it as given rather than aim at eliminating differential circumstances whenever possible. The difficulty can be put in terms internal to the EOp approach. In our economy status governs the return to effort, so that the same effort earns more for those born into high-status families than for those born into low-status ones: the reward schedule that is meant to certify an inequality as the product of responsible choice is itself shaped by a circumstance. Equalising welfare compensates for these unequal returns ex post while leaving the schedule that produces them intact, which is why an EOp ethic taken seriously cannot stop at redistributing income but must reach the structures that let a circumstance govern the productivity of effort.

To put it differently, in a static perspective, EOp requires us to equalise the opportunities of people living in slums as well as those living in suburban areas. But in a dynamic perspective, one may argue that the EOp ethic requires us to eliminate slums. This question can be ignored in a static framework and under the assumption that the equalising process inherent in the static EOp policy would also have a positive effect on circumstances. Our results show that this assumption may be seriously incorrect and the EOp ethic requires us to design policies aimed at changing circumstances. As Ruth Lister (2001) famously argued, equality of opportunity may be impossible to reach in the context of economic and social structures that remain profoundly unequal.

# 2 Model

## 2.1 The Economy

Consider an infinitely-lived economy in which each individual lives one period and has one offspring, so that population is constant over time. In every period $t = 1, 2, \ldots$, agents produce and consume a single good by exerting effort $e_t$. Each individual belongs to a social class depending on their income. Call the high-income social class, the *rich* ($r$) and the low-income class, the *poor* ($p$). Let $\alpha$ (resp., $1-\alpha$) be the proportion of poor (resp., rich) agents in the population, with $\alpha > 1/2$. Individual welfare depends positively on consumption, $c$ and negatively on effort, $e$:

$$u(c, e) = c - \frac{e^2}{2}, \tag{1}$$

The consumption level of an individual belonging to class $i$, $i = p, r$, is given by

$$c^i = (1-\tau_t)y_t^i + \tau_t[\alpha y_t^p + (1-\alpha)y_t^r], \tag{2}$$

where $\tau_t$ is a flat tax rate, $y_t^i$ is the pre-tax income of an individual belonging to class $i$, and $\alpha y_t^p + (1-\alpha)y_t^r$ is the average pre-tax income at $t$.

We assume that parental SES affects a child's opportunities in acquiring skills. Heckman (2013) argues that both cognitive and socio-emotional skills develop in early childhood through the family environment. Depending on the period of one's life this effect can take various forms. As Loury has noted,

> There are many reasons why a child's opportunities to acquire skills vary with the economic success of her parents. For example, the quality of schooling any child receives varies considerably across communities and tends to be higher in the suburbs than in the central city. Where there is housing segregation based on income, and the quality of neighbourhood schools shows a positive correlation with the community's wealth, a child's educational opportunities can be expected to vary directly with parental economic achievements. Further the absence of a perfect capital market for educational loans means that the opportunity for higher education and the quality of that education will be sensitive to an individual's socioeconomic background. (Loury, 1976, p. 155).

From an alternative (but complementary) viewpoint, status could capture the positive effects of social networks (or social capital). As Coleman (1988, 1990) argues, social capital plays the role of the effectiveness of human capital. In our model this is precisely the role status plays: it does not enter income additively but multiplies effort, so that social origin operates through the productivity of effort. Unlike a financial bequest, the advantage this confers cannot be transferred or taxed away directly, which will prove central to the policy implications we draw in the conclusion.

Of course, status per se does not generate earnings. What matters is the interplay of effort and status: effort is more or less productive depending on status. The conceptualisation of status as the productivity of effort is also consistent with a large empirical literature showing that social origins affect effort productivity (Hershbein, 2016; Chetty et al., 2016).

Formally, let $s_t^p$, $s_t^r \in \mathbb{R}_+$ denote the status of the rich and the poor, respectively. If status affects the incentives of individuals into exerting effort and hence can be conceptualised as the productivity of effort, then the income of an agent with status $s^i$ and exerting effort $e^i$ is given by $y_t^i = s_t^i e_t^i$, $i = p, r$.

Following a vast literature, we assume that status takes the form of a *positional good*, thus capturing one's *relative* socioeconomic standing (Hirsch, 1977; Besley and Ghatak, 2008; Moav and Neeman, 2010; Ray and Robson, 2012). The emphasis on status can be traced back to the works of Rae (1834), Veblen (1922) and Duesenberry (1949), who argued that individual consumption patterns are influenced by patterns in their reference groups. More recently, Frank (1985), Cole et al. (1992), Robson (1992), Clark and Oswald (1996), Corneo and Jeanne (1999), Becker et al. (2005), Moav and Neeman (2010), and Ray and Robson (2012) show how aiming to appear to have high status leads to conspicuous consumption motives which can lead to persistent inequality. We contribute to this literature by focusing on how status, acting as the productivity of effort, shapes the circumstances and choices of the next generation, and on the tension this creates for equality of opportunity.

We capture the positional nature of status by assuming that $s_t^p + s_t^r = 1$, for all $t$. For simplicity, let $s_t^p = s_t$ and $s_t^r = 1 - s_t$. We assume that $s_1 \in \left(0, \frac{1}{2}\right)$. Then, the pre-tax income of poor and rich agents is given, respectively, by:

$$y_t^p = s_t e_t^p, \tag{3}$$

$$y_t^r = (1 - s_t) e_t^r. \tag{4}$$

### Optimal Effort

Individuals choose their consumption and effort in order to maximise lifetime welfare, given their status and the tax rate. Formally, in every $t$, an agent from class $i = p, r$ solves

$$\max_{c_t^i, e_t^i} \left\{ c_t^i - \frac{(e_t^i)^2}{2} \right\},$$

subject to (2) and either (3) or (4). Then, the optimal effort of the two classes is

$$e_t^{p*} = (1 - \tau_t) s_t, \tag{5}$$

$$e_t^{r*} = (1 - \tau_t)(1 - s_t). \tag{6}$$

Hence, due to differences in status the rich have an incentive to exert more effort than the poor. Plugging (5) and (6) into (3) and (4), respectively, the optimal pre-tax income of the representative individual of each class is:

$$y_t^{p*} = (1 - \tau_t)(s_t)^2, \tag{7}$$

$$y_t^{r*} = (1 - \tau_t)(1 - s_t)^2. \tag{8}$$

Therefore, at the optimum, average income is $(1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]$. Given (2), (7), and (8), the consumption of the two classes can be expressed as follows

$$c_t^p = (1 - \tau_t)^2 (s_t)^2 + \tau_t (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2], \tag{9}$$

$$c_t^r = (1 - \tau_t)^2 (1 - s_t)^2 + \tau_t (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]. \tag{10}$$

Given (5), (6), (9), and (10), the indirect utilities of the two classes are

$$v^p(s_t, \tau_t) = \frac{(1 - \tau_t)^2 (s_t)^2}{2} + \tau_t (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2], \tag{11}$$

$$v^r(s_t, \tau_t) = \frac{(1 - \tau_t)^2 (1 - s_t)^2}{2} + \tau_t (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2]. \tag{12}$$

From (11)-(12), if $s_t < 1/2$, then it immediately follows that $v^p(s_t, \tau_t) \leq v^r(s_t, \tau_t)$ for all $\tau_t \in [0, 1]$, with equality holding only if $\tau_t = 1$.

Let $\tau_t^*$ denote the tax rate which maximises the utility of the worst off individual(s) in period $t$: in a static perspective, this would be the requirement of an equal opportunity ethics, given that $v^p(s_t, \tau_t) \leq v^r(s_t, \tau_t)$ for all tax rates. Formally:

$$\max_{\tau_t \in [0,1]} \left\{ \frac{(1 - \tau_t)^2 (s_t)^2}{2} + \tau_t (1 - \tau_t)[\alpha(s_t)^2 + (1 - \alpha)(1 - s_t)^2] \right\}. \tag{$MP$}$$

The following Lemma shows that this problem has a unique solution.[6]

**Lemma 1.** *For every* $s_t \in \left(0, \frac{1}{2}\right]$ *and* $\alpha \in \left(\frac{1}{2}, 1\right)$, $v^p(s_t, \cdot)$ *is strictly concave in* $\tau_t$, *and* $MP$ *has a unique solution, given by*

$$\tau_t^* = \frac{(1-\alpha)(1-2s_t)}{(s_t)^2 + 2(1-\alpha)(1-2s_t)}. \tag{13}$$

It is immediate to show that $\tau_t^*$ is decreasing in both $\alpha$ and $s_t$: *ceteris paribus*, higher status inequality (lower $s$) and a higher proportion of the rich in the population (lower $\alpha$) will both lead to a higher tax rate. The intuition is simple: if there are more rich agents, or they are more productive, then the loss in welfare for poor agents due to the disincentive effect of an increase in taxation is more than compensated by the redistributive effect.

## 2.2 Intergenerational Transmission of Status

An individual's status depends on her socio-economic background: the economic, cultural, and social conditions of her upbringing. In the context of our simple model, we focus on economic factors and suppose that the status of a generation depends on the after-tax income of their parents. To be specific, we assume

$$\frac{s_t^p}{s_t^r} = \frac{c_{t-1}^p}{c_{t-1}^r}. \tag{14}$$

Taking also into account the positional good nature of status, by (14) it follows that at $t$,

$$s_t = \frac{c_{t-1}^p}{c_{t-1}^r + c_{t-1}^p}, \tag{15}$$

while the status of the rich is $1 - s_t = \frac{c_{t-1}^r}{c_{t-1}^r + c_{t-1}^p}$. At the optimal $e_t$, the status of each class at $t+1$ is a function of the status of their parents and of the tax rate they faced in $t$:[7]

$$s_{t+1} = \frac{s_t^2 + \tau_t(1-\alpha)(1-2s_t)}{1 - 2s_t + 2s_t^2 + \tau_t(1-2s_t)(1-2\alpha)} \equiv f(s_t, \tau_t). \tag{16}$$

Thus, for all $\tau_t \in [0,1]$, if $s_t \in \left(0, \frac{1}{2}\right)$, then $s_{t+1} \in \left(0, \frac{1}{2}\right]$ (with $s_{t+1} = \frac{1}{2}$ only if $\tau_t = 1$), and $f(s_t, \tau_t)$ is strictly increasing in $\tau_t$ (see section B.3 of the Online Appendix). Moreover $f\left(\frac{1}{2}, \tau_t\right) = \frac{1}{2}$ for all $\tau_t \in [0,1]$, so $\left(0, \frac{1}{2}\right]$ is invariant under (16) and every status path generated by a policy from $s_1 \in \left(0, \frac{1}{2}\right)$ remains in it.[8]

The transmission rule (14) and the consumption levels (9)-(10) already contain the mech-

[6] The proofs of all formal results are in the Appendix.

[7] The complete derivation of equation (16) is in section B.2 of the Online Appendix.

[8] If $\tau_t = 1$ then optimal consumption is zero and the ratio $c_t^p/(c_t^p + c_t^r)$ is undefined. We set $f(s_t, 1) = \frac{1}{2}$, the common limit of (16) as $\tau_t \to 1$, which extends $f$ continuously to the closed feasible set $[0,1]$.

anism behind the results of Section 3. Since $c_t^i = v^i(s_t, \tau_t) + (e_t^i)^2/2$, the gap in consumption between the two classes decomposes as

$$c_t^r - c_t^p = [v^r(s_t, \tau_t) - v^p(s_t, \tau_t)] + \frac{1}{2}\left[(e_t^r)^2 - (e_t^p)^2\right], \tag{17}$$

and at the optimal effort levels (5)-(6) each term on the right equals $\frac{1}{2}(1-\tau_t)^2(1-2s_t)$. Half of the consumption gap is inequality in welfare; the other half is an *effort-cost gap* that arises solely because the rich, enjoying higher status, find effort more productive and supply more of it. Status is transmitted through relative consumption, so a policy that equalises welfare still leaves this second half in place, and it is this residual gap that reproduces the status differential across generations. It closes only as $\tau_t \to 1$, which is why, as we show below, no EOp policy brings status to its egalitarian value short of full confiscation. The very property that makes status a circumstance, its effect on the productivity of effort, is what prevents an EOp policy from removing it.

# 3 Equality of opportunity

## 3.1 Status as the only circumstance

What is the EOp programme in this context? Intuitively, it would seem natural to assume that, for each agent, both SES and date of birth are relevant circumstances. Yet, given that one of the key advantages of Roemer's approach is its political, rather than metaphysical, nature, this is not obvious. The definition of the set of circumstances is a political decision taken by the (opportunity) egalitarian polity and given that future people are not represented in today's political arena, it is important to explore the case where agents decide to ignore the dynamic implications of their decisions and focus on contemporary inequalities.

This is neither ethically arbitrary from the viewpoint of EOp theory, nor empirically groundless. Theoretically, a myopic focus is not necessarily the product of selfish behaviour: an egalitarian polity may well decide to focus on contemporary inequalities while leaving it to their descendants to decide how to deal with future inequalities. Furthermore, issues traditionally associated with intertemporal justice such as exhaustible resources, climate change, public debt and so on are excluded by fiat from our model. Empirically, there are examples of governments aiming to promote equality of opportunity mostly, or only, within a generation, as discussed in the Introduction.

When status is the only circumstance, the EOp tax rate maximises the welfare of the worst off in each $t$. Call this, the *Myopic* EOp (MEOp) programme. Let $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$ with $\tau_t \in [0, 1]$ all $t$ be an infinite sequence of tax rates. Noting that $v^p(s_t, \tau_t) \leq v^r(s_t, \tau_t)$ for all

$s_t \in (0, 1/2)$ and $\tau_t \in [0, 1]$, MEOp amounts to choosing $\boldsymbol{\tau}$ such that for all $t \geq 1$:

$$\max_{\tau_t \in [0,1]} v^p(s_t, \tau_t), \qquad \text{(MEOp)}$$

subject to (16), given $s_1$. A sequence $\boldsymbol{\tau} = \{\tau_t\}_{t=1}^{\infty}$ is called *a policy.*

Let $\boldsymbol{\tau^M} = \{\tau_t^M\}_{t=1}^{\infty}$ denote the solution of MEOp: given our assumptions, it exists and is unique, with $\tau_t^M = \tau_t^*$ all $t$. Further, at the solution of MEOp, $s_{t+1}$ is an increasing and strictly convex function of $s_t$.[9] The next result characterises the solutions of MEOp.

**Proposition 1.** *Let* $s_1 \in \left(0, \frac{1}{2}\right)$ *and* $s(\alpha) = \frac{1}{2}\left[3 - 2\alpha - \sqrt{4(\alpha-1)^2 + 1}\,\right]$.

*(i) At the solution of MEOp,* $s_t = s_1$ *all* $t$ *if and only if* $s_1 = s(\alpha) \in (0, \frac{1}{2})$.

*(ii) For* $s_1 \neq s(\alpha)$*, at the solution of MEOp,* $s_t$ *converges monotonically to* $s(\alpha)$*:* $\{s_t\}$ *is strictly increasing if* $s_1 < s(\alpha)$ *and strictly decreasing if* $s_1 > s(\alpha)$.

Proposition 1 has some puzzling implications for egalitarians. First, the MEOp policy will never lead to equality of income, not even in the limit. Indeed, if the economy starts off from a relatively egalitarian position ($s_1 > s(\alpha)$), the MEOp policy requires inequality to *grow* over time, approaching a potentially very high limit value.

If we assumed status to proxy also educational opportunities, then our result would be similar to Roemer and Unveren (2016) which proves the persistence of inequalities when education depends only on parental status. Unlike in the latter model, however, Proposition 1 also shows, and this is the second puzzling implication, that long-run inequalities depend on the relative population shares. At the solution of MEOp, a higher fraction of poor agents $\alpha$ leads to a higher level of long run inequality, that is, a lower $s(\alpha)$. A more polarised society but relatively equal economy, with a very small group of people who are only marginally better off than the rest of society, will eventually converge to a very large level of inequality *at the solution of MEOp.* For example, if $\alpha = 0.9$, then $s(0.9) \approx 0.0901$ but $s(0.99) \approx 0.0099$ and $s(0.999) \approx 0.00099$. With a one per cent elite, the MEOp policy drives the status of the remaining 99% down to about one per cent.

The intuition is simple. Taxation affects the welfare of individuals via two channels: on the one hand, high taxation means high redistribution, on the other hand, high taxation has a negative effect on individual incentives to exert effort. For simplicity call them, respectively, the *redistribution channel* and the *incentive channel.* Their relative strength depends on the population shares of the two classes. For a given level of status inequality, a high (low) proportion of poor individuals means that the benefits of the redistribution channel are relatively low (high) while the effects through the incentive channel are relatively high (low). More polarised societies with small groups of rich people will have lower levels of taxation, and therefore higher long-run inequality levels.

[9] The proof is in section B.4 of the Online Appendix.

At $t$, let $\tilde{\tau}_t = \tilde{\tau}_t(s_t)$ be the tax rate such that $s_{t+1} = s_t$. By equation (16), at $t$:

$$\tilde{\tau}_t = \frac{s_t - s_t^2}{1 - s_t - \alpha(1 - 2s_t)}, \tag{18}$$

and $\tilde{\tau}_t(s_t) \in [0, 1]$ for all $s_t \in \left(0, \frac{1}{2}\right)$.[10] Proposition 1 shows that for $s_t > s(\alpha)$ there exists a tension between maximising the utility of the poor and not diminishing their status. This can be further illustrated by the following.

**Lemma 2.** *At all $t$: (i) If $s_t = s(\alpha)$, then $\tilde{\tau}_t = \tau_t^M$. (ii) If $s_t > s(\alpha)$, then $\tilde{\tau}_t > \tau_t^M$. (iii) If $s_t < s(\alpha)$, then $\tilde{\tau}_t < \tau_t^M$.*

Lemma 2 highlights that if $s_t > s(\alpha)$, then the tax rate required for keeping status constant is higher than the one which maximises the utility of the poor. Of course, inequalities in status are important but one may argue that what ultimately matters to individuals is welfare. In order to examine the dynamics of welfare inequalities we substitute equation (13) into (11) to obtain the indirect utility of the poor at the solution of MEOp:

$$v^p(s_t) = \frac{[s_t^2 + (1 - \alpha)(1 - 2s_t)]^2}{2s_t^2 + 4(1 - \alpha)(1 - 2s_t)}. \tag{19}$$

Lemma 3 describes some properties of the indirect utility function of the poor.

**Lemma 3.** *For every $\alpha \in \left(\frac{1}{2}, 1\right)$, there exists a $\bar{s}(\alpha) \in (0, s(\alpha))$ such that $\frac{\partial v^p(s_t)}{\partial s_t} < 0$, for $s_t < \bar{s}(\alpha)$, and $\frac{\partial v^p(s_t)}{\partial s_t} > 0$, for $s_t > \bar{s}(\alpha)$.*

If the economy starts off with relatively limited inequality ($s_1 > s(\alpha)$), at the solution of MEOp the status of the poor will decrease over time and therefore their welfare will fall both in relative and by Lemma 3 *in absolute* terms. It can also be shown that, for all $\alpha \in \left(\frac{1}{2}, 1\right)$, $\bar{s}(\alpha)$ is decreasing in $\alpha$ and tends to 0 as $\alpha$ tends to 1.[11] Note that, since $s(\alpha) > \bar{s}(\alpha)$ and $v^p$ is increasing on $\left(s(\alpha), \frac{1}{2}\right)$, this implies that $v^p\left(s(\alpha)\right)$ is the infimum level of indirect utility of the MEOp for $s_1 > s(\alpha)$.

Let $\tilde{v}^p(s_t)$ be the indirect utility of the poor when the tax rate is $\tilde{\tau}_t$, so that status is constant at $s_t$. At $s_t = s(\alpha)$ the two policies coincide (Lemma 2(i)), so $\tilde{v}^p\left(s(\alpha)\right) = v^p\left(s(\alpha)\right)$. The behaviour of $\tilde{v}^p$ in a neighbourhood of $s(\alpha)$ will play a key role in the intergenerational analysis of the next subsection.

## 3.2 Date of birth and status as circumstances

The previous results show that a myopic egalitarian polity will implement a policy whose implications seem directly inconsistent with the EOp ethic. Of course, one may object that

[10] See section B.6 in the Online Appendix.
[11] See section B.5 in the Online Appendix.

this does not highlight a conceptual problem of Roemer's EOp approach: the troubling inegalitarian result is due to the fact that MEOp considers only class, and not date of birth, as a circumstance. This is disputable on ethical grounds: one's date of birth is beyond one's control. Indeed, one can take the previous results themselves as providing further support to this argument. Strictly speaking, this objection is not particularly cogent: if the approach is meant to be agnostic about the relevant circumstances, then its desirability cannot rely on the choice of a specific set of circumstances. However, in this section we consider the implications of considering date of birth as a circumstance, by extending MEOp to an *Intergenerational EOp* (IEOp) programme.

For every $\boldsymbol{\tau}$, $v^p(s_t, \tau_t) \leq v^r(s_t, \tau_t)$ for every $s_t \in \left(0, \frac{1}{2}\right]$ and $\tau_t \in [0,1]$: the poor are the worst off in each generation, and the infimum below is therefore the welfare floor of the economy as a whole, across classes and generations. The *welfare floor* of policy $\boldsymbol{\tau}$ is:

$$V_{\boldsymbol{\tau}}(s_1) := \inf_{t \geq 1} v^p(s_t, \tau_t).$$

The IEOp programme solves:

$$V(s_1) := \sup_{\boldsymbol{\tau} \in [0,1]^\infty} V_{\boldsymbol{\tau}}(s_1), \tag{IEOp}$$

and a policy $\boldsymbol{\tau}$ is the *optimal policy* if $V_{\boldsymbol{\tau}}(s_1) = V(s_1)$.

Theorem 1 bounds the value function $V$ on the whole state space, and characterises it exactly on part of it, for every $\alpha \in \left(\frac{1}{2}, 1\right)$.

**Theorem 1.** *Let* $\alpha \in \left(\frac{1}{2}, 1\right)$.

*(i)* $V(s_1) \leq v^p(s_1) < v^p\left(\frac{1}{2}\right) = \frac{1}{8}$ *for all* $s_1 \in \left(0, \frac{1}{2}\right)$.

*(ii) If* $s_1 \in [\bar{s}(\alpha), s(\alpha)]$, *then* $V(s_1) = v^p(s_1)$ *and* $\boldsymbol{\tau}^M$ *is the optimal policy. Further,* $s_t \leq s_{t+1}$ *all* $t$, $lim_{t\to\infty} s_t = s(\alpha)$, *and* $s_t \leq s(\alpha) < \frac{1}{2}$, *all* $t$.

*(iii) If* $s_1 \in \left(s(\alpha), \frac{1}{2}\right)$, *then*

$$\tilde{v}^p(s_1) \ \leq \ V(s_1) \ \leq \ v^p(s_1).$$

The status level $s(\alpha)$, at which the constant-status and myopic rates coincide (Lemma 2), is a cut-off. Below it, on $[\bar{s}(\alpha), s(\alpha))$, the myopic policy raises status towards $s(\alpha)$, so the planner can do no better than maximising each generation's welfare, and the value is the myopic $v^p(s_1)$. Above it the two policies diverge, and two simple ones bracket the optimum: holding status at $s_1$ forever secures every generation the constant-status welfare $\tilde{v}^p(s_1)$, while no policy can give even the first generation more than its first-period maximum value $v^p(s_1)$. (Clearly, no policy can undo the circumstance inherited by generation 1.)

The value of IEO$_p$ welfare floor lies strictly below the egalitarian level $v^p\left(\frac{1}{2}\right) = \frac{1}{8}$ from every initial condition (part (i)) and the economy does not reach an equal status. The myopic policy in (ii) converges only to $s(\alpha) < \frac{1}{2}$, while the stationary policy securing the lower bound in (iii) holds status fixed at $s_1 < \frac{1}{2}$.[12] Thus the EOp policy does not undo inequality and the inequality it sustains is not the product of responsible choices. Persistent inequality is consistent with the IEOp policy while remaining inconsistent with the EOp ethic, which would require eliminating the status differential itself.

For $s_1 \in (0, \bar{s}(\alpha))$, the difficulty is the non-monotonicity of $v^p$ in Lemma 3: the myopic ascent towards $s(\alpha)$ must first cross the welfare trough at $\bar{s}(\alpha)$, so the worst-off generation need no longer be the first, and the argument of part (ii) fails. Two bounds are nonetheless immediate. Theorem 1(i) gives $V(s_1) \leq v^p(s_1)$; and under the myopic policy each generation receives $v^p(s_t) \geq v^p(\bar{s}(\alpha))$, the global minimum of the one-period value (Lemma 3), so $V(s_1) \geq v^p(\bar{s}(\alpha))$. Numerically, this floor exceeds the constant-status value $\tilde{v}^p(s_1)$, so that holding status fixed is dominated in this region; and for $s_1$ small the upper bound is strict: attaining it would require $\tau_1 = \tau^*(s_1)$, the unique maximiser of first-period welfare (Lemma 1), which sends status to a point of strictly lower one-period value, so that generation 2 falls short of $v^p(s_1)$. How the crossing of the trough is best managed remains open. We note, however, that the region $(0, \bar{s}(\alpha))$ is rather marginal, especially if $\alpha$ is large.

# 4 Concluding discussion

We have analysed opportunity-egalitarian policies in an economy where the inequality of one generation, operating through status as a positional good, shapes the marginal returns to effort, and hence the circumstances, of the next. Proposition 1 and Theorem 1 deliver an unpalatable conclusion for the political approach to EOp (Roemer, 1996, 1998; Roemer et al., 2003): an opportunity-egalitarian policy does not in general reduce inequality.

The myopic policy drives an initially more equal economy up towards an ever more unequal steady state, and even the intergenerational policy, which need not let the welfare of the poor decline, cannot sustain the welfare of full equality from any starting point. The more polarised the society, the more unequal the myopic steady state. In no case does the EOp policy take the economy to equality, even though the inequalities it sustains are *not* the product of individual responsible choices: persistent inequality is consistent with EOp policy while remaining inconsistent with the EOp ethic.

It may be objected that the model omits many features of actual economies. But its purpose is to make a theoretical point, not to represent social reality in full, and the point is general: it concerns an ambiguity in the definition of circumstances once the EOp problem is placed in a dynamic context. A second objection is that the results depend on the restriction to linear taxation, and that progressive or type-dependent schemes would yield different

[12] For $s_1 \in \left(s(\alpha), \frac{1}{2}\right)$, the intergenerational planner strictly improves on the myopic policy, whose value is exactly $v^p\left(s(\alpha)\right)$ in this region. For a proof, see section B.7.

conclusions. This is true but, within a pragmatic political approach to EOp, beside the point: linear taxation is standard in the literature, type-dependent schemes may be politically infeasible (Roemer et al., 2003), and the approach cannot tell us a priori which to prefer, since the feasible set is shaped by the political process rather than by normative considerations.

Roemer conceives of his theory as a micro-oriented approach, to be applied "after society has put in place its macro policies which address the alleviation of disadvantage" (Roemer 1998, p.114). One reading of our results is therefore as a cautionary tale about the micro approach, whose feasible policies in any specific context are necessarily limited. We think the issue is more fundamental. There is no reason why the normative foundations of macro EOp policies should differ from those of specific contexts, and the same algorithm is applied by Roemer (1998) at the societal level. In our model there is no micro/macro distinction, and even there the EOp policy conflicts with the EOp ethic, under which "equality of opportunity for X holds when the values of X for all those who exercised a comparable degree of responsibility are equal, regardless of their circumstances" (Roemer 1993, p.149).

Theoretically, our results call for a reconsideration of the notion of circumstance in a dynamic context: once a circumstance governs the return to effort, equalising outcomes within a period does not equalise the circumstance, and may allow inequality to grow. At the policy level, they point to two correctives of different depth. The first stays within redistribution but changes its target: since status is transmitted through relative after-tax income, a scheme that equalises that income rather than within-period welfare acts on the channel of transmission itself, and progressive taxation is the natural instrument, desirable not only because it brings the economy closer to the EOp ethic within a period but because its dynamic effects work towards the eventual elimination of man-made differences in status.

The second corrective goes beyond redistribution altogether: the differential we identify originates in social capital governing the productivity of effort, which no redistribution of income removes, and addressing it requires acting on the structures that make social standing and networks pay off per unit of effort. This is more demanding and more faithful to the EOp ethic, which would equalise not the fruits of a circumstance but the circumstance itself.

## References


Arneson, R., 1989. Equality and and equality of opportunity for welfare. Philosophical Studies 56, 77–93.

Atkinson, A.B., Piketty, T., Saez, E., 2011. Top incomes in the long run of history. Journal of Economic Literature 49, 3–71.

Becker, G.S., Murphy, K.M., Werning, I., 2005. The Equilibrium Distribution of Income and the Market for Status. Journal of Political Economy 113, 282–310.

Bénabou, R., Tirole, J., 2006. Belief in a Just World and Redistributive Politics. The Quarterly Journal of Economics 121, 699–746.

Besley, T., Ghatak, M., 2008. Status Incentives. American Economic Review 98, 206–211.

Bossert, W., 1995. Redistribution mechanisms based on individual characteristics. Mathematical Social Sciences 29, 1 – 17.

Bourguignon, F., Ferreira, F.H.G., Menéndez, M., 2007. Inequality of opportunity in brazil. Review of Income and Wealth 53, 585–618.

Checchi, D., Peragine, V., 2005. Regional Disparities and Inequality of Opportunity: The Case of Italy. IZA Discussion Papers 1874. Institute of Labor Economics (IZA).

Chetty, R., Grusky, D., Hell, M., Hendren, N., Manduca, R., Narang, J., 2016. The Fading American Dream: Trends in Absolute Income Mobility Since 1940. NBER Working Papers 22910. National Bureau of Economic Research, Inc.

Clark, A.E., Oswald, A.J., 1996. Satisfaction and comparison income. Journal of Public Economics 61, 359–381.

Cohen, G.A., 1989. On the currency of egalitarian justice. Ethics 99, 906?944.

Cole, H.L., Mailath, G.J., Postlewaite, A., 1992. Social Norms, Savings Behavior, and Growth. Journal of Political Economy 100, 1092–1125.

Coleman, J.S., 1988. Social Capital in the Creation of Human Capital. American Journal of Sociology 94, S95–S120.

Coleman, J.S., 1990. Equality and Achievement in Education. Westview Press, Boulder, CO.

Corneo, G., Jeanne, O., 1999. Pecuniary Emulation, Inequality and Growth. European Economic Review 43, 1665–1678.

Department of Social Security, 1999. Opportunity for all: Tackling poverty and social exclusion. First annual report.

Duesenberry, J., 1949. Income, Saving and the Theory of Consumer Behavior. Oxford University Press, New York.

Dworkin, R., 1981. What is equality? part 2: Equality of resources. Philosophy and Public Affairs 10, 283–345.

Fleurbaey, M., 1995. Three solutions for the compensation problem. Journal of Economic Theory 65, 505– 521.

Fleurbaey, M., 2008. Fairness, Responsibility and Welfare. Oxford University Press.

Fleurbaey, M., Maniquet, F., 2011. A Theory of Fairness and Social Welfare. Cambridge University Press.

Frank, R., 1985. Choosing the Right Pond: Human Behavior and the quest for status. Oxford University Press, New York.

Van de gaer, D., 1993. Equality of opportunity and investment in human capital. PhD Thesis, KULeuven.

Van de gaer, D., Schokkaert, E., Martinez, M., 2001. Three Meanings of Intergenerational Mobility. Economica 68, 519–538.

Heckman, J.J., 2013. Giving Kids a Fair Chance (A Strategy That Works). MIT Press, Cambridge, MA.

Hershbein, B., 2016. A College Degree is worth Less if you are Raised Poor. Social Mobility Memos. Brookings.

Hills, J., Stewart, K.e., 2005. A more equal society? New Labour, poverty, inequality and exclusion. The Policy Press, Bristol, UK.

Hirsch, F., 1977. Social Limits to Growth. Routledge and Kegan Paul, London.

Lister, R., 2001. Doing good by stealth. New Economy 8, 65–70.

Loury, G.C., 1976. A Dynamic Theory of Racial Income Differences. Discussion Papers 225. Northwestern University, Center for Mathematical Studies in Economics and Management Science.

Milanovic, B., 2016. Global Inequality: A New Approach for the Age of Globalization. Harvard University Press, Cambridge, MA.

Moav, O., Neeman, Z., 2010. Status and Poverty. Journal of the European Economic Association 8, 413–420.

Piketty, T., 1995. Social Mobility and Redistributive Politics. The Quarterly Journal of Economics 110, 551–84.

Piketty, T., 1998. Self-fulfilling beliefs about social status. Journal of Public Economics 70, 115–132.

Piketty, T., 2014. Capital in the Twenty-First Century. Harvard University Press, Cambridge, MA.

Rae, J., 1834. The Sociological Theory of Capital. MacMillan, New York.

Ramos, X., Van de gaer, D., 2016. Approaches to inequality of opportunity: Principles, measures and evidence. Journal of Economic Surveys 30, 855–883.

Rawls, J., 1971. A Theory of Justice. Belknap Press of Harvard University Press.

Ray, D., Robson, A., 2012. Status, Intertemporal Choice, and Risk-Taking. Econometrica 80, 1505–1531.

Robson, A.J., 1992. Status, the Distribution of Wealth, Private and Social Attitudes to Risk. Econometrica 60, 837–857.

Roemer, J.E., 1996. Theories of Distributive Justice. Harvard University Press.

Roemer, J.E., 1998. Equality of Opportunity. Harvard University Press.

Roemer, J.E., Aaberge, R., Colombino, U., Fritzell, J., Jenkins, S.P., Lefranc, A., Marx, I., Page, M., Pommer, E., Ruiz-Castillo, J., San Segundo, M.J., Tranaes, T., Trannoy, A., Wagner, G.G., Zubiri, I., 2003. To what extent do fiscal regimes equalize opportunities for income acquisition among citizens? Journal of Public Economics 87, 539 – 565.

Roemer, J.E., Trannoy, A., 2015. Equality of opportunity, in: Atkinson, A.B., Bourguignon, F. (Eds.), Handbook of Income Distribution: Volume 2A. North Holland, Amsterdam, pp. 217–300.

Roemer, J.E., Trannoy, A., 2016. Equality of opportunity: Theory and measurement. Journal of Economic Literature 54.

Roemer, J.E., Unveren, B., 2016. Dynamic equality of opportunity. Economica , 322–343.

Roemer, J.E., Veneziani, R., 2004. What We Owe Our Children, They Their Children,. . . . Journal of Public Economic Theory 6, 637–654.

Saez, E., Zucman, G., 2019. The Triumph of Injustice: How the Rich Dodge Taxes and How to Make Them Pay. W. W. Norton, New York.

Stiglitz, J.E., 2012. The Price of Inequality: How Today's Divided Society Endangers Our Future. W. W. Norton, New York.

Veblen, T., 1922. The Theory of the Leisure Class. An Economic Study of Institutions. George Allen Unwin, London. (First published, 1899).

Zucman, G., 2024. A Blueprint for a Coordinated Minimum Effective Taxation Standard for Ultra-High-Net-Worth Individuals. Technical Report. EU Tax Observatory.

# A Appendix: Proofs

**Proof of Lemma 1:**

From (11),

$$\frac{\partial v^p(s_t,\tau_t)}{\partial \tau_t} = -(1-\tau_t)s_t^2 + (1-2\tau_t)\left[\alpha s_t^2 + (1-\alpha)(1-s_t)^2\right],$$

$$\frac{\partial^2 v^p(s_t,\tau_t)}{\partial \tau_t^2} = s_t^2 - 2\left[\alpha s_t^2 + (1-\alpha)(1-s_t)^2\right] = -s_t^2 - 2(1-\alpha)(1-2s_t) < 0.$$

Strict concavity follows, and the first-order condition has the unique root $\tau_t^*$ given in (13). This root is feasible: the numerator of (13) is nonnegative and its denominator exceeds twice the numerator by $s_t^2 > 0$, so $\tau_t^* \in \left[0, \frac{1}{2}\right) \subset [0,1]$ with $\tau_t^* = 0$ only at $s_t = \frac{1}{2}$. □

**Proof of Lemma 2:**

(i) follows from the definition of the stationary state. (ii) Both $\tilde{\tau}_t$ and $\tau_t^M$ are continuous, and so is their difference. It is sufficient to show that $\tilde{\tau}_t - \tau_t^M = 0$ has only one real solution for $s_t \in [0, \frac{1}{2}]$ and that for some $s_t > s(\alpha)$, $\tilde{\tau}_t > \tau_t^M$. The solutions of $\tilde{\tau}_t - \tau_t^M = 0$ are

1. $s = 1-\alpha-\sqrt{\alpha^2-\alpha}$,
2. $s = 1-\alpha+\sqrt{\alpha^2-\alpha}$,
3. $s = \frac{1}{2}\left[3-2\alpha+\sqrt{4(\alpha-1)^2+1}\right]$,
4. $s = \frac{1}{2}\left[3-2\alpha-\sqrt{4(\alpha-1)^2+1}\right]$ $(= s(\alpha))$.

The first two are complex numbers while the third does not lie within $(0, \frac{1}{2})$. Note that $s(\alpha)$ is decreasing in $\alpha$, hence on $\left(\frac{1}{2}, 1\right)$ it is bounded above by its supremum as $\alpha \downarrow \frac{1}{2}$, namely $s\left(\frac{1}{2}\right) = 1 - \frac{1}{\sqrt{2}} < \frac{1}{3}$; so $s(\alpha) < \frac{1}{3}$ for all $\alpha \in \left(\frac{1}{2}, 1\right)$. Then for $s_t = \frac{1}{3}$

$$\tilde{\tau}_t - \tau_t^M = \frac{-4+15\alpha-9\alpha^2}{42-57\alpha+18\alpha^2},$$

which is greater than zero as both the numerator and the denominator are positive for $\alpha \in (\frac{1}{2}, 1)$. (iii) For $s \to 0$, $\tilde{\tau}_t - \tau_t^M = -\frac{1}{2} < 0$. □

**Proof of Lemma 3:**

We first show that for $s_t \in \left[s(\alpha), \frac{1}{2}\right)$, $\frac{\partial v^p(s_t)}{\partial s_t} > 0$.

Differentiating (19) we obtain

$$\frac{\partial v^p(s_t)}{\partial s_t} = \frac{A_1 A_2}{[s_t^2 + 2(1-\alpha)(1-2s_t)]^2}, \tag{20}$$

where $A_1 = 1 - \alpha - 2s_t + 2\alpha s_t + s_t^2 = (1-\alpha)(1-2s_t) + s_t^2 > 0$ and

$$A_2 = -2 + 4\alpha - 2\alpha^2 + 7s_t - 11\alpha s_t + 4\alpha^2 s_t - 6s_t^2 + 6\alpha s_t^2 + s_t^3.$$

Hence, it is sufficient to prove that also $A_2 > 0$ for all $s_t \in \left[s(\alpha), \frac{1}{2}\right)$. In order to prove this we show that (a) $\frac{\partial A_2}{\partial s_t} = 7 - 11\alpha + 4\alpha^2 - 12s_t + 12\alpha s_t + 3s_t^2 > 0$ for all $\alpha \in \left(\frac{1}{2}, 1\right)$ and that (b) for $s_t = s(\alpha)$, $A_2 > 0$

(a) Let $F(\alpha, s_t) := 7 - 11\alpha + 4\alpha^2 - 12s_t + 12\alpha s_t + 3s_t^2$. Then,

$$\frac{\partial F}{\partial \alpha} = 0 \Leftrightarrow -11 + 8\alpha + 12s_t = 0, \tag{21}$$

$$\frac{\partial F}{\partial s_t} = 0 \Leftrightarrow -12 + 12\alpha + 6s_t = 0. \tag{22}$$

From (21) and (22) we get $\alpha = \frac{13}{16}$ and $s = \frac{3}{8}$. The second partial derivatives are $\frac{\partial^2 F}{\partial \alpha^2} = 8$, $\frac{\partial^2 F}{\partial s_t^2} = 6$, $\frac{\partial^2 F}{\partial s_t \partial \alpha} = 12$. Hence, the determinant of the Hessian of $F(\alpha, s_t)$ is strictly negative and $(13/16,\ 3/8)$ is a non degenerate saddle point. Because $F\left(\frac{13}{16}, \frac{3}{8}\right) > 0$, in order to prove that $F(\alpha, s_t) > 0$ for all $\alpha \in \left(\frac{1}{2}, 1\right)$ and $s_t \in \left(s(\alpha), \frac{1}{2}\right)$ it is sufficient to show that the three dimensional graph of $F(\alpha, s_t)$ takes non negative values at the edges of the parallelogram of $\alpha, s_t$. The following equations represent these edges:

$$F(1, s_t) = 7 - 11 + 4 - 12s_t + 12s_t + 3s_t^2$$

$$F(1/2, s_t) = 7 - 5.5 + 1 - 6s_t + 3s_t^2 = 2.5 - 6s_t + 3s_t^2$$

$$F(\alpha, 0) = 7 - 11\alpha + 4\alpha^2 = (1-\alpha)(7-4\alpha)$$

$$F(\alpha, 1/2) = 7/4 - 5\alpha + 4\alpha^2$$

Clearly, $F(1, s_t) \geq 0$ for all $s_t \in [0, \frac{1}{2}]$ and $F(\alpha, 0) > 0$ for all $\alpha \in \left[\frac{1}{2}, 1\right]$. Then, note that $F(\frac{1}{2}, s_t)$ is decreasing in $s_t$ over $(0, \frac{1}{2})$ thus it is minimised for $s_t = 1/2$, but $F(1/2, 1/2) = -0.5 + 0.75 > 0$. Finally, $F(\alpha, \frac{1}{2})$ is a parabola and reaches a minimum for $\alpha = 5/8$, for which $F(5/8, 1/2) > 0$.

(b) At $s_t = s(\alpha)$,

$$A_2 = \frac{1}{2}(2\alpha - 1)\left[7 + 4\alpha^2 - 10\alpha + (2\alpha - 3)\sqrt{5 - 8\alpha + 4\alpha^2}\right] > 0$$

if and only if $7 + 4\alpha^2 - 10\alpha > (3 - 2\alpha)\sqrt{5 - 8\alpha + 4\alpha^2}$. After some algebra the latter expression simplifies to $4(\alpha - 1)^2 > 0$ which is always true.

Since (a) and (b) are true, then $A_2 > 0$ for all $s_t \in \left[s(\alpha), \frac{1}{2}\right)$.

To conclude the proof, observe that $A_2$ is increasing in $s_t \in \left(0, \frac{1}{2}\right)$, $A_2 < 0$ for $s_t = 0$, and $A_2 > 0$ for $s_t = s(\alpha)$. Therefore given that $A_2$ is a continuous function of $s_t$, there exists $\bar{s}(\alpha) \in (0, s(\alpha))$, such that $A_2 = 0$ for $s_t = \bar{s}(\alpha)$. $\square$

**Proof of Proposition 1:**

Substituting (13) into (16), at the solution of MEOp,

$$s_{t+1} - s_t = \frac{1}{2}\left[1 - 2s_t + \frac{2s_t - 1}{3 + 2s_t(s_t - 3) + 2\alpha(2s_t - 1)}\right]. \tag{23}$$

(i) At a stationary solution, $s_{t+1} = s_t$. By (23), it follows that $s_{t+1} = s_t$ for $s_t = \frac{1}{2}$ and at the solutions of $s^2 + s(2\alpha - 3) + 1 - \alpha = 0$, namely,

$$s_{\pm}(\alpha) = \frac{1}{2}\left[3 - 2\alpha \pm \sqrt{4(1-\alpha)^2 + 1}\,\right].$$

Because $\alpha \in \left(\frac{1}{2}, 1\right)$, it follows that $3 - 2\alpha > \sqrt{4(1-\alpha)^2 + 1} > 1$, $s_+(\alpha) > 1$, and $s_-(\alpha) = s(\alpha) \in (0, \frac{1}{2})$.

(ii) As there exists only one solution of $s_{t+1} - s_t = 0$ for $s_t \in (0, \frac{1}{2})$, namely $s(\alpha)$, and $s_{t+1} - s_t \to \frac{1}{2}\left[1 - \frac{1}{3-2\alpha}\right] > 0$ as $s_t \to 0$, continuity of $s_{t+1} - s_t$ gives $s_{t+1} - s_t > 0$ on $(0, s(\alpha))$; and since $s(\alpha)$ is its only zero in $(0, \frac{1}{2})$, its sign is constant on $(s(\alpha), \frac{1}{2})$, where evaluating (23) at an interior point gives $s_{t+1} - s_t < 0$. Since, in addition, $s_{t+1}$ is an increasing function of $s_t$ at the solution of MEOp (section B.4 of the Online Appendix), with fixed point $s(\alpha)$, the orbit remains on one side of $s(\alpha)$: increasing and bounded above by $s(\alpha)$ for $s_1 \in (0, s(\alpha))$, decreasing and bounded below by it for $s_1 \in \left(s(\alpha), \frac{1}{2}\right)$. It therefore converges, and by continuity its limit is a fixed point of the dynamics in $\left(0, \frac{1}{2}\right)$, hence $s(\alpha)$: for $s_1 \neq s(\alpha)$, at the solution of MEOp, $s_t$ converges to $s(\alpha)$. $\square$

**Proof of Theorem 1:**

For any policy $\boldsymbol{\tau}$, since the first generation is included in the welfare floor and $\tau_1 \in [0, 1]$,

$$V_{\boldsymbol{\tau}}(s_1) = \inf_{t \geq 1} v^p(s_t, \tau_t) \leq v^p(s_1, \tau_1) \leq \max_{\tau \in [0,1]} v^p(s_1, \tau) = v^p(s_1),$$

where the last equality holds by Lemma 1 and (19). Taking the supremum over all policies gives

$$V(s_1) \leq v^p(s_1). \tag{24}$$

For the strict inequality in part (i), note that the denominator of (19) is strictly positive on $\left[0, \frac{1}{2}\right]$, so $v^p$ extends continuously to this interval, with $v^p(0) = \frac{1-\alpha}{4}$ and $v^p\left(\frac{1}{2}\right) = \frac{1}{8}$ by direct evaluation. By Lemma 3, $v^p$ is strictly decreasing on $(0, \bar{s}(\alpha))$ and strictly increasing on $\left(\bar{s}(\alpha), \frac{1}{2}\right)$; a continuous function with this shape takes values strictly below the larger of its two endpoint values at every interior point. Since $\alpha > \frac{1}{2}$ gives $v^p(0) = \frac{1-\alpha}{4} < \frac{1}{8} = v^p\left(\frac{1}{2}\right)$, it follows that $v^p(s_1) < v^p\left(\frac{1}{2}\right) = \frac{1}{8}$ for every $s_1 \in \left(0, \frac{1}{2}\right)$, which proves part (i).

Now let $s_1 \in [\bar{s}(\alpha), s(\alpha)]$ and consider the myopic policy $\boldsymbol{\tau^M}$, given by $\tau_t = \tau^M(s_t)$ for all $t$. Proposition 1 implies that the status sequence generated by this policy is nondecreasing and converges to $s(\alpha)$: it is constant at $s(\alpha)$ if $s_1 = s(\alpha)$, by part (i), and strictly increasing towards $s(\alpha)$ if $s_1 \in [\bar{s}(\alpha), s(\alpha))$, by part (ii). Hence

$$s_t \in [s_1, s(\alpha)] \subseteq \left[\bar{s}(\alpha), \tfrac{1}{2}\right) \qquad \text{for all } t.$$

By Lemma 3, $v^p$ is increasing on $\left[\bar{s}(\alpha), \frac{1}{2}\right)$. Under $\boldsymbol{\tau^M}$, generation $t$ receives $v^p\Big(s_t, \tau^M(s_t)\Big) = v^p(s_t)$, the reduced utility (19); therefore every generation receives at least the first-period myopic welfare:

$$v^p(s_t) \geq v^p(s_1) \qquad \text{for all } t,$$

with equality at $t = 1$. The welfare floor of the myopic policy is thus $V_{\boldsymbol{\tau^M}}(s_1) = v^p(s_1)$, so $V(s_1) \geq v^p(s_1)$. Together with (24), this gives $V(s_1) = v^p(s_1)$; hence $V_{\boldsymbol{\tau^M}}(s_1) = V(s_1)$, so $\boldsymbol{\tau^M}$ is optimal, which proves the optimality and path properties stated in part (ii).

Finally, let $s_1 \in \left(s(\alpha), \frac{1}{2}\right)$. The upper bound in part (iii) is exactly (24). For the lower bound, take the stationary policy $\tilde{\boldsymbol{\tau}}$, given by $\tau_t = \tilde{\tau}(s_1)$ for all $t$; this is a policy, since $\tilde{\tau}(s_1) \in [0, 1]$ by section B.6 of the Online Appendix. Since $\tilde{\tau}(s_1)$ is the tax rate that holds status constant, $s_t = s_1$ for all $t$. Every generation therefore receives

$$v^p\Big(s_1, \tilde{\tau}(s_1)\Big) = \tilde{v}^p(s_1),$$

so this policy has welfare floor $V_{\tilde{\boldsymbol{\tau}}}(s_1) = \tilde{v}^p(s_1)$ and hence $V(s_1) \geq \tilde{v}^p(s_1)$. Combining this lower bound with (24) gives

$$\tilde{v}^p(s_1) \leq V(s_1) \leq v^p(s_1),$$

with the lower bound attained by the stationary policy, whose status path satisfies $s_t = s_1 < \frac{1}{2}$ for all $t$.

$\square$

# B Online Appendix

## B.1 Proof that $\tau^*$ is decreasing in $\alpha$ and $s$

The derivative with respect to $\alpha$ is:

$$\frac{\partial \tau}{\partial \alpha} = \frac{-(1-2s)[s^2 + 2(1-\alpha)(1-2s)] + 2(1-2s)(1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2},$$

which simplifies to

$$\frac{\partial \tau}{\partial \alpha} = \frac{-(1-2s)s^2}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

Also, the derivative with respect to $s$ is:

$$\frac{\partial \tau}{\partial s} = \frac{-2(1-\alpha)[s^2 + 2(1-\alpha)(1-2s)] - [2s - 4(1-\alpha)](1-\alpha)(1-2s)}{[s^2 + 2(1-\alpha)(1-2s)]^2},$$

or

$$\frac{\partial \tau}{\partial s} = \frac{-2s(1-s)(1-\alpha)}{[s^2 + 2(1-\alpha)(1-2s)]^2} < 0.$$

□

## B.2 Status at optimal effort level

From equations (10), (9) and (15), the status at $t+1$ is

$$s_{t+1} = \frac{(1-\tau_t)^2 s_t^2 + \tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}{(1-\tau_t)^2[(1-s_t)^2 + s_t^2] + 2\tau_t(1-\tau_t)[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]},$$

or

$$s_{t+1} = \frac{(1-\tau_t)s_t^2 + \tau_t[\alpha s_t^2 + (1-\alpha)(1-s_t)^2]}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[\alpha s_t^2 + (1-\alpha)(1-2s_t+s_t^2)]},$$

or

$$s_{t+1} = \frac{s_t^2 - \tau_t s_t^2 + \tau_t \alpha s_t^2 + \tau_t(1-s_t)^2 - \tau_t\alpha(1-s_t)^2}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[\alpha s_t^2 + 1 - 2s_t + s_t^2 - \alpha + 2\alpha s_t - \alpha s_t^2]},$$

or

$$s_{t+1} = \frac{s_t^2 - \tau_t s_t^2 + \tau_t \alpha s_t^2 + \tau_t(1-s_t)^2 - \tau_t\alpha(1-s_t)^2}{(1-\tau_t)(1-2s_t+2s_t^2) + 2\tau_t[1 - 2s_t + s_t^2 - \alpha + 2\alpha s_t]},$$

or

$$s_{t+1} = \frac{s_t^2 + \tau_t(1-\alpha)(1-2s_t)}{1 - 2s_t + 2s_t^2 + \tau_t(1-2s_t)(1-2\alpha)}.$$

□

## B.3 Monotonicity of $f$ in $\tau$

Write $D(s,\tau) = 1-2s+2s^2+\tau(1-2s)(1-2\alpha)$ for the denominator of $f$ in (16). Differentiating,

$$\frac{\partial f(s,\tau)}{\partial \tau} = \frac{(1-2s)\,[s^2-2(1-\alpha)s+(1-\alpha)]}{D(s,\tau)^2}.$$

Fix $s \in \left(0,\frac{1}{2}\right)$ and $\alpha \in \left(\frac{1}{2},1\right)$. Then $1-2s>0$, and the bracket $s^2-2(1-\alpha)s+(1-\alpha)$, viewed as a quadratic in $s$, has discriminant $4(1-\alpha)^2-4(1-\alpha) = -4\alpha(1-\alpha) < 0$ and positive leading coefficient, hence is strictly positive. The denominator $D(s,\tau)^2$ is nonnegative, and $D(s,\tau) > 0$ on $\tau \in [0,1]$: $D$ is affine in $\tau$ with $D(s,0) = 1-2s+2s^2 > 0$ and $D(s,1) = 2(1-s)^2-2\alpha(1-2s)$, which is decreasing in $\alpha$ and so bounded below by its value at $\alpha = 1$, namely $2s^2 > 0$. Therefore $\frac{\partial f}{\partial \tau} > 0$: for every $s \in \left(0,\frac{1}{2}\right)$ and $\alpha \in \left(\frac{1}{2},1\right)$, $f(s,\cdot)$ is strictly increasing on $[0,1]$. $\square$

## B.4 Proof that at the solution to MEOp $s_{t+1}$ is increasing and strictly convex in $s_t$

Substituting (13) into (16), at the solution of MEOp,

$$s_{t+1} = \frac{(1-s_t)^2-\alpha(1-2s_t)}{2s_t^2+(3-2\alpha)(1-2s_t)}.$$

Then

$$\frac{\partial s_{t+1}}{\partial s_t} = \frac{2s_t(1-s_t)}{(3-2\alpha-6s_t+4\alpha s_t+2s_t^2)^2} > 0,$$

and

$$\frac{\partial^2 s_{t+1}}{\partial s_t^2} = -\frac{2(-3+2\alpha+6s_t^2-4s_t^3)}{[(1-2s_t)(3-2\alpha)+2s_t^2]^3}.$$

Given that the denominator of the latter expression is positive, $\frac{\partial^2 s_{t+1}}{\partial s_t^2} > 0$ if and only if $-3+2\alpha+6s_t^2-4s_t^3 < 0$. The result then follows noting that $6s_t^2-4s_t^3$ is bounded above by 1 for all $s_t \in [0,\frac{1}{2}]$. $\square$

## B.5 Proof that $\bar{s}(\alpha)$ is decreasing in $\alpha$

The exact value of $\bar{s}(\alpha)$ is given by the solution of $A_2 = 0$. In order to prove (i) it is sufficient to prove that the inverse of $\bar{s}(\alpha)$ is decreasing in $s$ For simplicity call the inverse $\bar{\alpha}$ such that

$$\bar{\alpha} = \bar{s}^{-1}(\alpha). \tag{25}$$

Then $\bar{\alpha}$ is given by the solution of

$$-2+4\alpha-2\alpha^2+7s_t-11\alpha s_t+4\alpha^2 s_t-6s_t^2+6\alpha s_t^2+s_t^3 = 0, \tag{26}$$

which is between 0 and 1/2. (26) has two solutions

1. $\alpha = \frac{-4+11s_t-6s^2-s_t\sqrt{9-28s_t+20s_t^2}}{4(2s_t-1)}$ and

2. $\alpha = \frac{-4+11s_t-6s^2+s_t\sqrt{9-28s_t+20s_t^2}}{4(2s_t-1)}$,

only the second of which is between 0 and 1/2. Hence

$$\bar{\alpha} = \frac{-4+11s_t-6s_t^2+s_t\sqrt{9-28s_t+20s_t^2}}{4(2s_t-1)}. \tag{27}$$

In order to prove the first part of the corollary, we need to show that

$$\frac{d\bar{\alpha}}{ds_t} = \frac{20s_t^2+3\left(3+\sqrt{9-28s_t+20s_t^2}\right)-6s_t\left(4+\sqrt{9-28s_t+20s_t^2}\right)}{4(2s_t-1)\sqrt{9-28s_t+20s_t^2}} < 0. \tag{28}$$

Note that the denominator of the above is negative hence it is sufficient to prove that the numerator is positive. Given that $s_t < 1/2$,

$$3\sqrt{9-28s_t+20s_t^2} > 6s_t\sqrt{9-28s_t+20s_t^2},$$

thus it is sufficient to prove that

$$20s_t^2+9-24s_t > 0.$$

$20s_t^2+9-24s_t$ is decreasing in $s_t$, hence minimised for $s_t = 1/2$,which means that

$$20(1/2)^2+9-24(1/2) = 2 > 0,$$

which concludes the first part of the corollary. Note that for $s_t = 1$, $\bar{\alpha} = 1$. Hence given continuity and the fact that $\bar{\alpha}$ is decreasing in $s_t$, proves the second part of the corollary. □

## B.6 Constant inequality tax rate

The first property of the constant inequality tax rate we need to show is that $\tilde{\tau}_t(s_t) = \frac{s_t-s_t^2}{1-s_t-\alpha(1-2s_t)} \in [0,1]$. Note that $s_t - s_t^2 < 1$ given that $s_t \in \left(0, \frac{1}{2}\right)$ and also

$$1-s_t-\alpha(1-2s_t) = 1-\alpha+s_t(2\alpha-1) > 0,$$

due to $\alpha > \frac{1}{2}$. Hence $\tilde{\tau}_t(s_t) > 0$. For $\tilde{\tau}_t(s_t) \leq 1$, it is sufficient to show that

$$s_t - s_t^2 < 1-s_t-\alpha(1-2s_t),$$

or,

$$-s_t^2 < (1-2s_t)(1-\alpha),$$

which is true for all $s_t \in \left(0, \frac{1}{2}\right)$. $\square$

## B.7 Proof that if $s_1 \in \left(s(\alpha), \frac{1}{2}\right)$, the intergenerational planner strictly improves on the myopic policy

We prove that for every $\alpha \in \left(\frac{1}{2}, 1\right)$, there exists $\delta > 0$ such that $\tilde{v}^p(s) > v^p\left(s(\alpha)\right)$ for all $s \in (s(\alpha), s(\alpha)+\delta)$.

By Lemma 2(i), $\tilde{\tau}(s(\alpha)) = \tau^M(s(\alpha))$, so $\tilde{v}^p\left(s(\alpha)\right) = v^p\left(s(\alpha)\right)$. Along the MEOp path $v^p(s) = v^p(s, \tau^M(s))$ with $\tau^M(s)$ the interior maximiser, so $\frac{\partial v^p(s,\tau^M(s))}{\partial \tau} = 0$ (Lemma 1) and, by the envelope theorem, $\frac{dv^p(s)}{ds} = \frac{\partial v^p(s,\tau^M(s))}{\partial s}$. Along the constant-status path $\tilde{v}^p(s) = v^p(s, \tilde{\tau}(s))$,

$$\frac{d\tilde{v}^p(s)}{ds} = \frac{\partial v^p(s,\tilde{\tau}(s))}{\partial s} + \frac{\partial v^p(s,\tilde{\tau}(s))}{\partial \tau}\frac{d\tilde{\tau}(s)}{ds}.$$

At $s = s(\alpha)$ the cross term vanishes, since $\tilde{\tau}(s(\alpha)) = \tau^M(s(\alpha))$ makes $\partial_\tau v^p = 0$ there, and so

$$\frac{d\tilde{v}^p(s(\alpha))}{ds} = \frac{\partial v^p\Big(s(\alpha), \tau^M(s(\alpha))\Big)}{\partial s} = \frac{dv^p(s(\alpha))}{ds}.$$

By Lemma 3, $s(\alpha) > \bar{s}(\alpha)$, so $\frac{dv^p(s(\alpha))}{ds} = \frac{d\tilde{v}^p(s(\alpha))}{ds} > 0$, and by continuity there exists $\delta > 0$ such that $\tilde{v}^p$ is strictly increasing on $(s(\alpha), s(\alpha)+\delta)$, where $\tilde{v}^p(s) > \tilde{v}^p\left(s(\alpha)\right) = v^p\left(s(\alpha)\right)$, which proves the claim. $\square$

The restriction to a neighbourhood matters: freezing status at an arbitrary level need not improve on the myopic floor, since in very polarised societies $\tilde{v}^p(s_1)$ can fall below $v^p\left(s(\alpha)\right)$ (see the discussion of the region $s_1 \in (0, \bar{s}(\alpha))$ in the main text).

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